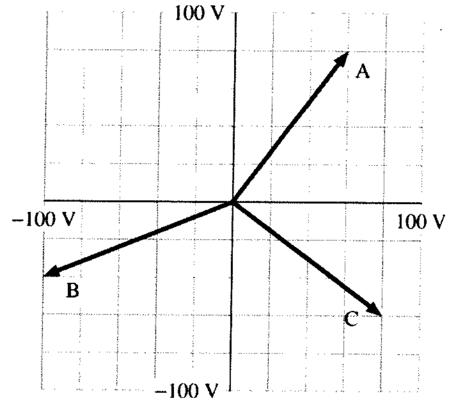
32 AC Circuits

32.1 AC Sources and Phasors

1. The figure shows emf phasors A, B, and C.



   a. What is the instantaneous value of the emf?

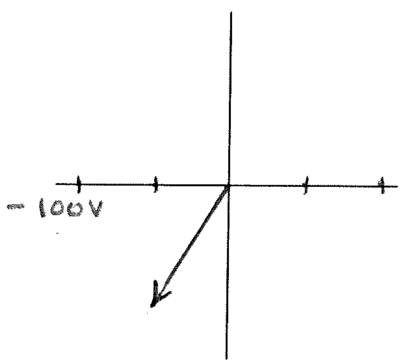
   A 60V B −100V C 80V

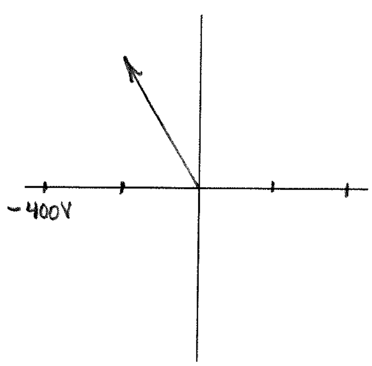
   b. At this instant, is the magnitude of the emf increasing, decreasing, or holding constant?

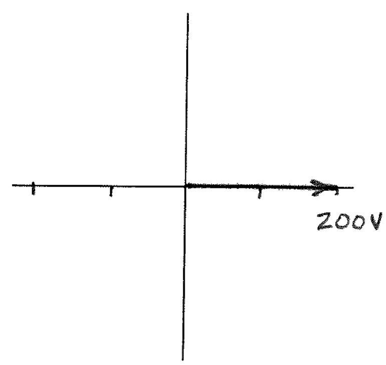
   A Decreasing B Decreasing C Increasing

2. Draw a phasor diagram for the following emfs.

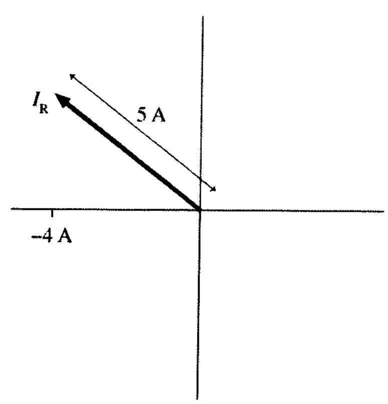
   a. $(100\text{ V})\cos\omega t$ at $\omega t = 240°$ b. $(400\text{ V})\cos\omega t$ at $t = \frac{1}{3}T$ c. $(200\text{ V})\cos\omega t$ at $t = 0$







3. The current phasor is shown for a 10 Ω resistor.



   a. What is the instantaneous resistor voltage v_R?

   $v_R = i_R R = (-4\text{A})(10\Omega) = -40\text{V}$

   b. What is the peak resistor voltage V_R?

   $V_R = I_R R = (5\text{A})(10\Omega) = 50\text{V}$

4. A circuit consists of one resistor connected to an emf. The peak current through the resistor is 4.0 A. What is the peak current if:

a. The resistance R is doubled?

2.0 A $\quad I_R = \frac{V_R}{R}$

b. The peak emf $\mathcal{E}_0$ is doubled?

8.0 A $\quad I_R = \frac{V_R}{R} = \frac{\mathcal{E}_0}{R}$

c. The frequency ω is doubled?

2.0A $\quad I_R$ is independent of frequency

32.2 Capacitor Circuits

5. A circuit consists of one capacitor connected to an emf. The peak current through the capacitor is 4.0 A. What is the peak current if:

a. The peak emf $\mathcal{E}_0$ is doubled?

8.0 A $\quad I_C = \omega C V_C = \omega C \mathcal{E}_0$

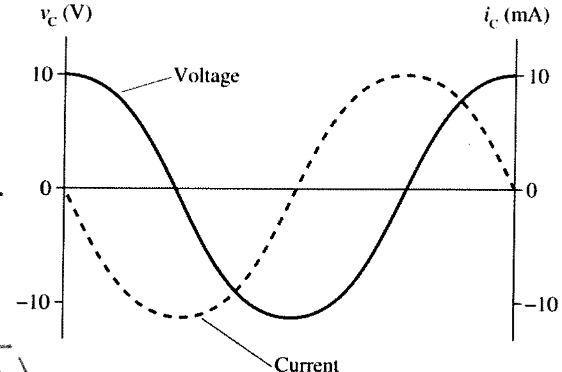
b. The capacitance C is doubled?

8.0 A

c. The frequency ω is doubled?

8.0 A

6. Current and voltage graphs are shown for a capacitor circuit with $\omega = 1000$ rad/s.



a. What is the capacitive reactance X_C?

$$X_C = \frac{V_C}{I_C} = \frac{10\text{V}}{0.01\text{A}} = 1000\,\Omega$$

b. What is the capacitance C?

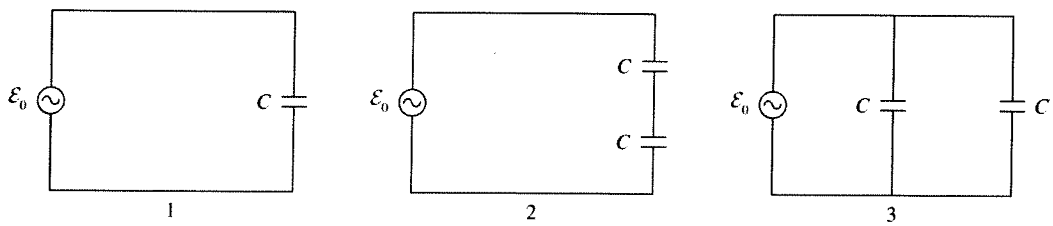
$$C = \frac{1}{\omega X_C} = \frac{1}{(1000\,\frac{\text{rad}}{\text{s}})(1000\,\Omega)}$$

$$C = 1.0 \times 10^{-6}\,\text{F} = 1\,\mu\text{F}$$

7. A 13 μF capacitor is connected to a 5.5 V/250 Hz oscillator. What is the instantaneous capacitor current i_C when $\mathcal{E} = -5.5$ V?

Zero. When $\mathcal{E} = -\mathcal{E}_0$, the voltage phasor angle is 180°. The current phasor leads the voltage phasor by 90° so it is at 270° which gives $i_C = 0$ since $\cos(270°) = 0$.

8. Consider these three circuits.



Rank in order, from largest to smallest, the peak currents $(I_C)_1$ to $(I_C)_3$ provided by the emf.

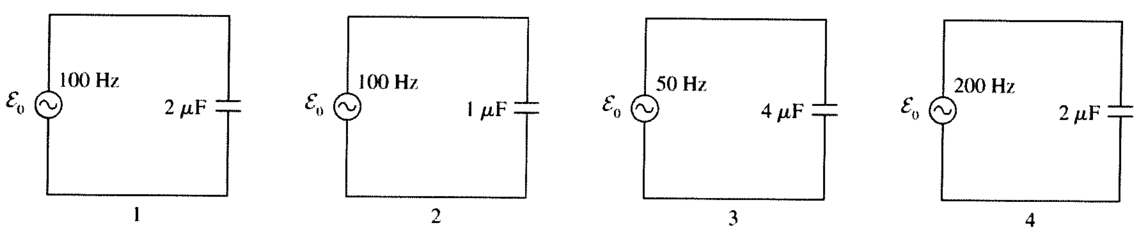
Order: $(I_C)_3 > (I_C)_1 > (I_C)_2$

Explanation:

$I_C = \omega C V_C$

$C_1 = C \quad C_2 = \frac{1}{2} C \quad C_3 = 2C$

9. Consider these four circuits.



Rank in order, from largest to smallest, the capacitive reactances $(X_C)_1$ to $(X_C)_4$.

Order: $(X_c)_2 > (X_c)_1 = (X_c)_3 > (X_c)_4$

Explanation:

$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ so $X_c \propto \frac{1}{fC}$

Smaller values of fC give larger X_c.

$f_1 C_1 = (100\,\text{Hz})(2\,\mu\text{F}) = 200\,\mu\text{F/s}$

$f_2 C_2 = 100\,\mu\text{F/s} \quad f_3 C_3 = 200\,\mu\text{F/s} \quad f_4 C_4 = 400\,\mu\text{F/s}$

32.3 *RC* Filter Circuits

10. A low-pass *RC* filter has a crossover frequency $f_c = 200$ Hz. What is f_c if:

a. The resistance *R* is halved?

$f_c = 400$ Hz since $f_c = \frac{1}{2\pi}\left(\frac{1}{RC}\right)$.

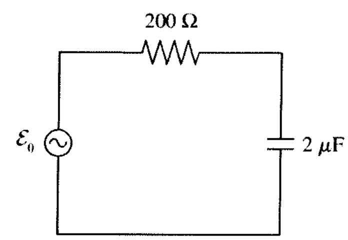
b. The capacitance *C* is halved?

$f_c = 400$ Hz

c. The peak emf $\mathcal{E}_0$ is halved?

$f_c = 200$Hz since f_c is independent of $\mathcal{E}_0$.

11. What new resistor value *R* will give this circuit the same value of ω_c if the capacitor value is changed to:

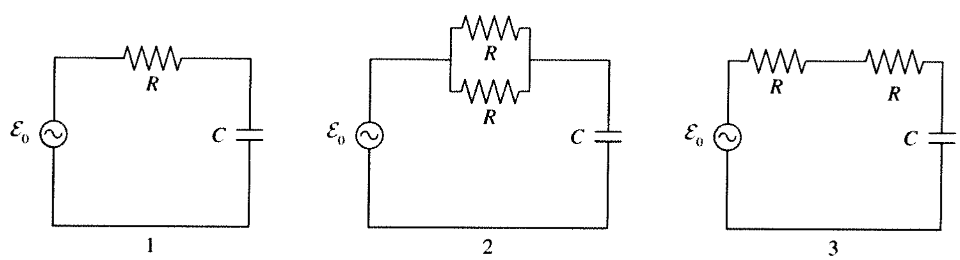


a. $C = 1\ \mu$F $R =$ 400 Ω

b. $C = 4\ \mu$F $R =$ 100 Ω

c. $C = 20\ \mu$F $R =$ 20 Ω

12. Consider these three circuits.



Rank in order, from largest to smallest, the crossover frequencies ω_{c1} to ω_{c3}.

Order: $\omega_{c2} > \omega_{c1} > \omega_{c3}$

Explanation:

$\omega_{c1} = \frac{1}{RC}$ $\omega_{c2} = \frac{1}{\left(\frac{R}{2}\right)C} = 2\,\omega_{c1}$

$\omega_{c3} = \frac{1}{(2R)C} = \frac{1}{2}\,\omega_{c1}$

13. The text claims that $V_R = V_C = \mathcal{E}_0/\sqrt{2}$ at $\omega = \omega_c$. If this is true, then $V_R + V_C > \mathcal{E}_0$. Is it possible for their sum to be larger than $\mathcal{E}_0$? Explain.

Yes. In fact, $V_R + V_C$ is always larger than $\mathcal{E}_0$ in an RC circuit (where both R and C are nonzero). V_R and V_C are two sides of a right triangle with $\mathcal{E}_0$ as the hypotenuse ($\mathcal{E}_0^2 = V_R^2 + V_C^2$). Also, the peak voltages do not occur at the same time.

32.4 Inductor Circuits

14. A circuit consists of one inductor connected to an emf. The peak current through the inductor is 4.0 A. What is the peak current if:

a. The peak emf $\mathcal{E}_0$ is doubled?

$I_L = 8.0\text{A}$ since $I_L = \frac{V_L}{X_L} = \frac{\mathcal{E}_0}{\omega L}$

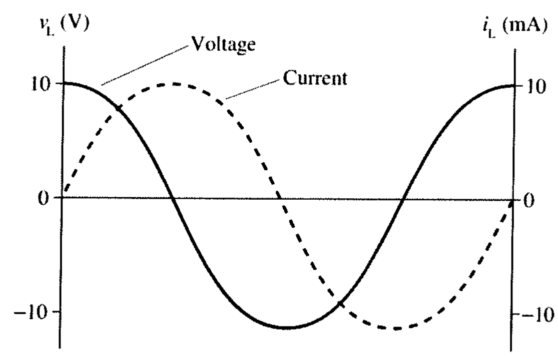
b. The inductance L is doubled?

$I_L = 2.0\text{A}$

c. The frequency ω is doubled?

$I_L = 2.0\text{A}$

15. Current and voltage graphs are shown for an inductor circuit with $\omega = 1000$ rad/s.



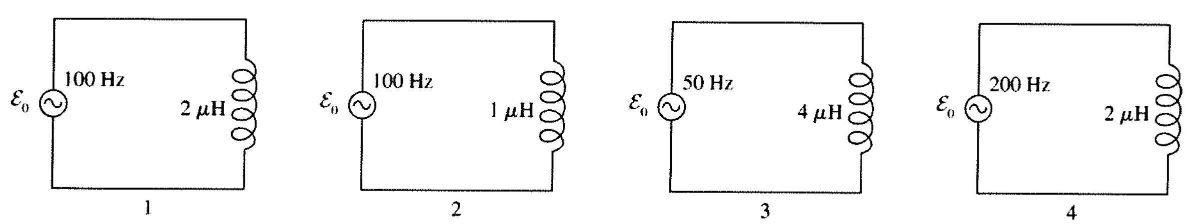
a. What is the inductive reactance X_L?

$X_L = \frac{V_L}{I_L} = \frac{10\text{V}}{10 \times 10^{-3}\text{A}} = 1000\,\Omega$

b. What is the inductance L?

$L = \frac{X_L}{\omega} = \frac{1000\,\Omega}{1000\text{ rad/s}} = 1.0\text{H}$

16. Consider these four circuits.



Rank in order, from largest to smallest, the inductive reactances $(X_L)_1$ to $(X_L)_4$.

Order: $(X_L)_4 > (X_L)_1 = (X_L)_3 > (X_L)_2$

Explanation:

$X_L = \omega L = 2\pi f L$ so $X_L \propto fL$ Larger fL implies larger X_L

$f_1 L_1 = (100\text{Hz})(2\mu\text{H}) = 200\,\mu\text{H/s}$

$f_2 L_2 = 100\,\mu\text{H/s}$, $f_3 L_3 = 200\,\mu\text{H/s}$, $f_4 L_4 = 400\,\mu\text{H/s}$

32.5 The Series *RLC* Circuit

17. The resonance frequency of a series *RLC* circuit is 1000 Hz. What is the resonance frequency if:

a. The resistance *R* is doubled?

1000 Hz since $\omega_0 = \frac{1}{\sqrt{LC}}$ is independent of R.

b. The inductance *L* is doubled?

$\frac{1}{\sqrt{2}} 1000\,\text{Hz} = 707\,\text{Hz}$

c. The capacitance *C* is doubled?

$\frac{1}{\sqrt{2}} 1000\,\text{Hz} = 707\,\text{Hz}$

d. The peak emf $\mathcal{E}_0$ is doubled?

1000 Hz since ω_0 is independent of $\mathcal{E}_0$

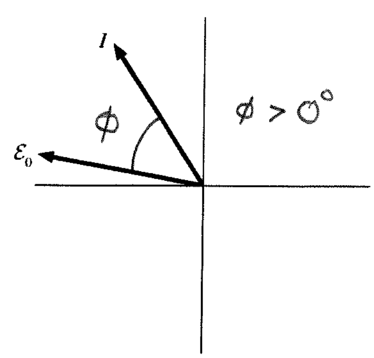
e. The frequency ω is doubled?

1000 Hz since ω_0 is independent of ω.

18. For these combinations of resistance and reactance, is a series *RLC* circuit in resonance (Yes or No)? Does the current lead the emf, lag the emf, or is it in phase with the emf?

R	X_L	X_C	Resonance?	Current?
100 Ω	100 Ω	50 Ω	No.	Lags.
100 Ω	50 Ω	100 Ω	No.	Leads.
100 Ω	75 Ω	75 Ω	Yes.	In phase.

19. In this series *RLC* circuit, is the emf frequency less than, equal to, or greater than the resonance frequency ω_0? Explain.



Greater than. Here current lags the emf, so $X_L > X_C$. By definition,

$X_L = \omega L$ and $X_C = \frac{1}{\omega C}$ so

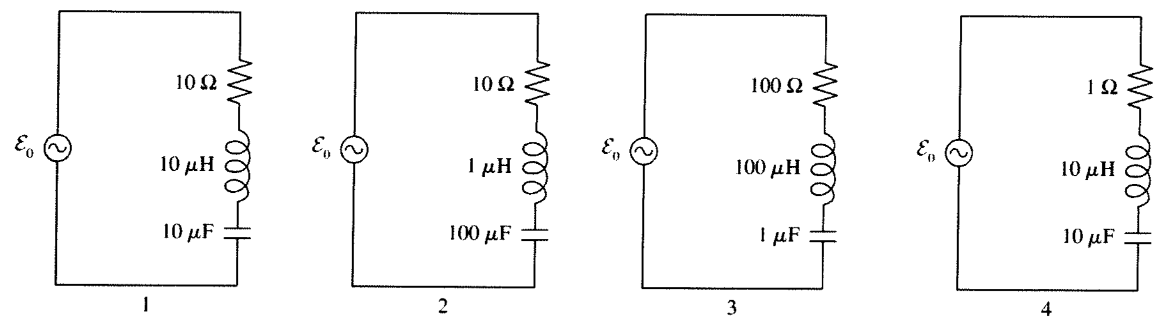
$$\frac{X_L}{X_C} = \omega^2 LC > 1 \Rightarrow \omega > \frac{1}{\sqrt{LC}} = \omega_0$$

$$\boxed{\omega > \omega_0}$$

20. The emf frequency of a series *RLC* circuit is less than the circuit's resonance frequency. Does the current lead or lag the emf? Explain.

Given $\omega < \omega_0$, then the reactances must have values such that $X_L < X_C$ so the phase angle ϕ between the emf and the current is less than zero. The current leads the emf.

21. Consider these four circuits. They all have the same resonance frequency ω_0.



Rank in order, from largest to smallest, the maximum currents $(I_{max})_1$ to $(I_{max})_4$.

Order: $(I_{max})_4 > (I_{max})_1 = (I_{max})_2 > (I_{max})_3$

Explanation:

$(I_{max})_1 = \frac{\mathcal{E}_0}{R} = \frac{\mathcal{E}_0}{10\,\Omega}$ $\quad$ $(I_{max})_2 = \frac{\mathcal{E}_0}{10\,\Omega} = (I_{max})_1$

$(I_{max})_3 = \frac{\mathcal{E}_0}{100\,\Omega} = \frac{(I_{max})_1}{10}$ $\quad$ $(I_{max})_4 = \frac{\mathcal{E}_0}{1\,\Omega} = 10(I_{max})_1$

32.6 Power in AC Circuits

22. The current in a series *RLC* circuit lags the emf by 20°. You cannot change the emf. What two different things could you do to the circuit that would increase the power delivered to the circuit by the emf?

The power will increase when the current increases.
To increase current:

1) reduce resistance R in the circuit.
2) choose L and C components such that their reactances are more closely equal, $X_L = X_C$. This will more closely match the frequency of the given emf with the resonant frequency of the circuit.

23. An average power dissipated by a resistor in an all-resistor circuit is 4.0 W. What is P_{avg} if:

a. The resistance R is doubled?

2.0 W since $P_{avg} = \frac{(V_{rms})^2}{R}$

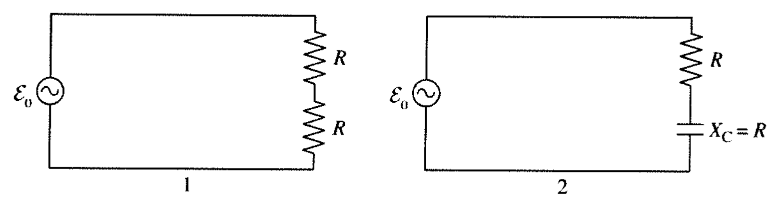
b. The peak emf $\mathcal{E}_0$ is doubled?

16.0 W since $P_{avg} = \frac{(V_{rms})^2}{R} = \frac{(\mathcal{E}_{rms})^2}{R} = \frac{(\mathcal{E}_0/\sqrt{2})^2}{R}$

c. The frequency is doubled?

4.0 W since P_{avg} is independent of frequency.

24. Consider these two circuits.



a. Which has the larger peak current? Or do both have the same peak current? Explain.

The peak current is larger in circuit 2.

Circuit 1: $I_{peak} = \frac{\mathcal{E}_0}{2R}$

Circuit 2: $I_{peak} = \frac{\mathcal{E}_0}{Z} = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_C^2}} = \frac{\mathcal{E}_0}{\sqrt{R^2+R^2}} = \frac{\mathcal{E}_0}{\sqrt{2}R}$

b. Which emf supplies the larger power? Or do both supply the same power?

Both circuits supply the same power.

Circuit 1: $P_{source} = I_{rms}\mathcal{E}_{rms} = \frac{I_{peak}}{\sqrt{2}}\frac{\mathcal{E}_0}{\sqrt{2}} = \frac{\mathcal{E}_0}{2\sqrt{2}R}\frac{\mathcal{E}_0}{\sqrt{2}} = \frac{\mathcal{E}_0^2}{4R}$

Circuit 2: $P_{source} = I_{rm}\mathcal{E}_{rms}\cos\phi$

where $\phi = \tan^{-1}\left(\frac{-X_c}{R}\right) = -45°$ so $\cos\phi = \frac{1}{\sqrt{2}}$

$P_{source} = \frac{I_{peak}}{\sqrt{2}}\frac{\mathcal{E}_0}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{\mathcal{E}_0}{2R}\frac{\mathcal{E}_0}{\sqrt{2}}\frac{1}{\sqrt{2}} = \frac{\mathcal{E}_0^2}{4R}$