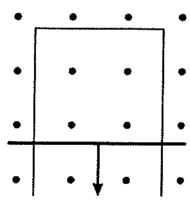
30 Electromagnetic Induction

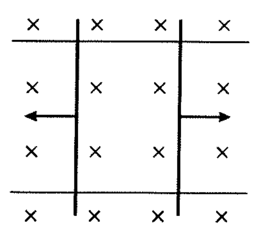
30.1 Induced Currents

30.2 Motional emf

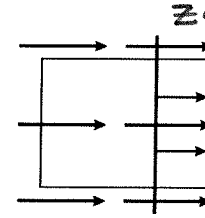
1. The figures below show one or more metal wires sliding on fixed metal rails in a magnetic field. For each, determine if the induced current flows clockwise, flows counterclockwise, or is zero. Show your answer by drawing it.

a.


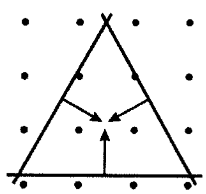
CW

b.


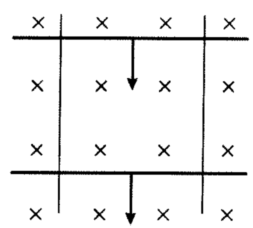
CCW

c.


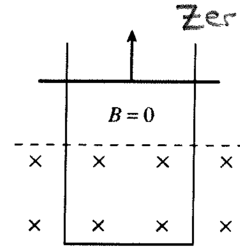
Zero

d.


CCW

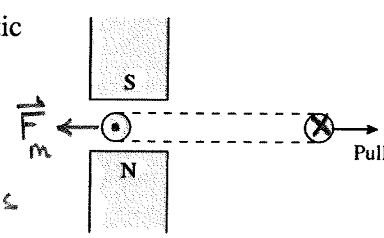
e.


Zero

f.


Zero

2. A loop of copper wire is being pulled from between two magnetic poles.



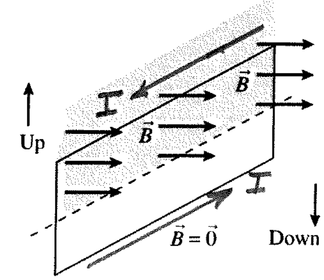
a. Show on the figure the current induced in the loop. Explain your reasoning.

The side of the loop between the poles is in a magnetic field that points up. This length of the loop is moving right so the force on the charge carriers in the wire is out of the page.

b. Does either side of the loop experience a magnetic force? If so, draw and label a vector arrow or arrows on the figure to show any forces.

Yes, to the left (opposite the pull).

3. A vertical, rectangular loop of copper wire is half in and half out of a horizontal magnetic field. (The field is zero beneath the dashed line.) The loop is released and starts to fall.



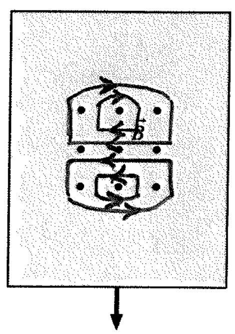
   a. Add arrows to the figure to show the direction of the induced current in the loop.

   b. Is there a net magnetic force on the loop? If so, in which direction? Explain.

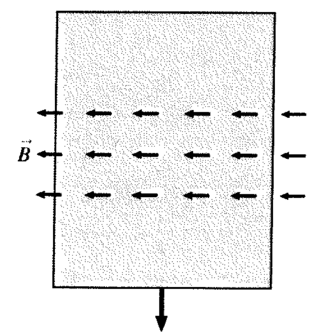
Yes. The net magnetic force is up. Only the top side experiences a magnetic force, and the direction of this force is found by $I\vec{\ell} \times \vec{B}$. (Note, the loop accelerates down due to a larger downward force, gravity).

4. Two very thin sheets of copper are pulled through a magnetic field. Do eddy currents flow in the sheet? If so, show them on the figures, with arrows to indicate the direction of flow. If not, why not?

a.

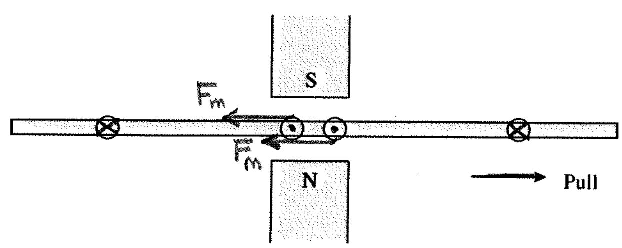


b.



No eddy currents since the magnetic force on the charges is perpendicular to the plane.

5. The figure shows an edge view of a copper sheet being pulled between two magnetic poles.



   a. Add a dot or an × to each of the circles to indicate the direction in which eddy currents are flowing in and out of the page.

   b. Do the currents you labeled in part a experience magnetic forces? If so, add force vectors to the figure to show the directions. If not, why not?

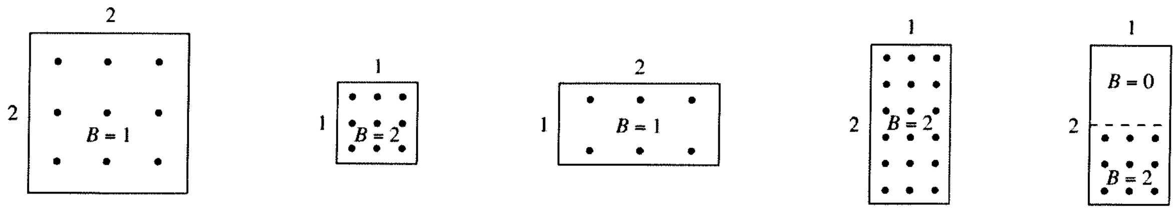
Both currents pointing out of the page experience a magnetic force to the right ($I\vec{\ell} \times \vec{B}$). The currents pointing into the page have essentially no magnetic force on them since the field from the poles is too weak there.

   c. Is there a net magnetic force on the copper sheet? If so, in which direction?

Yes, to the left.

30.3 Magnetic Flux

6. The figure shows five loops in a magnetic field. The numbers indicate the lengths of the sides and the strength of the field. Rank in order, from largest to smallest, the magnetic fluxes Φ_1 to Φ_5. Some may be equal.



Loop 1 Loop 2 Loop 3 Loop 4 Loop 5

Order: $\Phi_1 = \Phi_4 > \Phi_2 = \Phi_3 = \Phi_5$

Explanation:

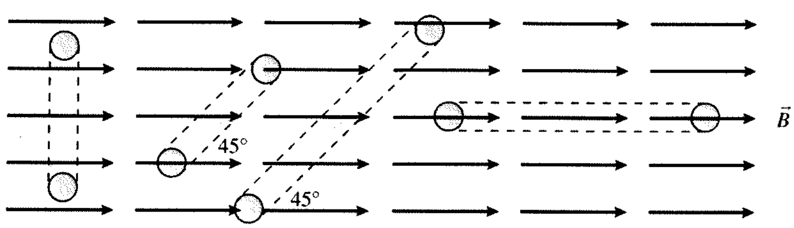
$\Phi = A_{eff} B$

$\Phi_1 = 2 \cdot 2 \cdot 1 = 4$ $\Phi_2 = 1 \cdot 1 \cdot 2 = 2$

$\Phi_3 = 1 \cdot 2 \cdot 1 = 2$ $\Phi_4 = 2 \cdot 1 \cdot 2 = 4$

$\Phi_5 = 1 \cdot 1 \cdot 2 = 2$

7. The figure shows four circular loops that are perpendicular to the page. The radius of loops 3 and 4 is twice that of loops 1 and 2. The magnetic field is the same for each. Rank in order, from largest to smallest, the magnetic fluxes Φ_1 to Φ_4. Some may be equal.



Loop 1 $r = 1$ Loop 2 $r = 1$ Loop 3 $r = 2$ Loop 4 $r = 2$

Order: $\Phi_3 > \Phi_1 > \Phi_2 > \Phi_4$

Explanation:

$\Phi = AB\cos\Theta = (\pi r^2) B \cos\Theta$

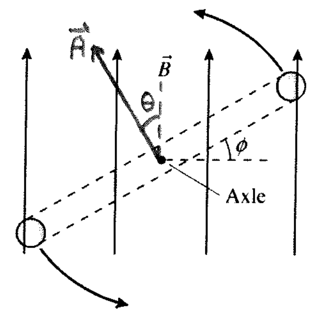
$\Phi_1 = \pi (1)^2 B \cos 0° = \pi B$

$\Phi_2 = \pi (1)^2 B \cos 45° = 0.707\, \Phi_1$

$\Phi_3 = \pi (2)^2 B \cos 45° = 2.83\, \Phi_1$

$\Phi_4 = \pi (2)^2 B \cos 90° = 0$

8. A circular loop rotates at constant speed about an axle through the center of the loop. The figure shows an edge view and defines the angle ϕ, which increases from 0° to 360° as the loop rotates.



a. At what angle or angles is the magnetic flux a maximum?

0° and 180°

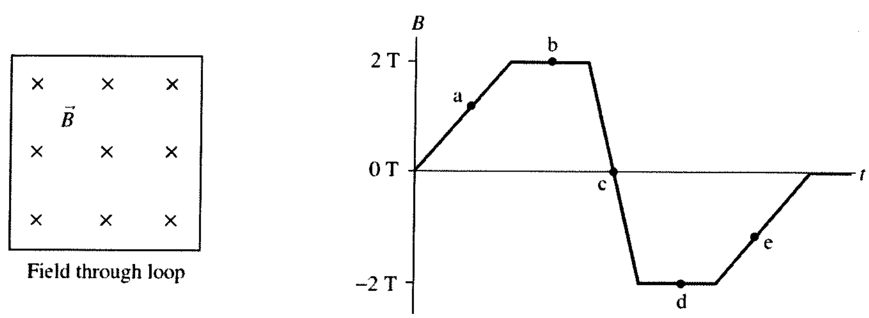
b. At what angle or angles is the magnetic flux a minimum?

90° and 270°

c. At what angle or angles is the magnetic flux *changing* most rapidly? Explain your choice.

At 90° and 270°. The flux is $\Phi = AB\cos\theta = AB\cos\phi$, and $\cos\phi$ has the most rapid change (steepest slope) at 90° and 270°.

9. A magnetic field is perpendicular to a loop. The graph shows how the magnetic field changes as a function of time, with positive values for B indicating a field into the page and negative values a field out of the page. Several points on the graph are labeled.



a. At which lettered point or points is the flux through the loop a maximum?

b and d, using the magnitude of flux.

b. At which lettered point or points is the flux through the loop a minimum?

c. Zero flux.

c. At which point or points is the flux changing most rapidly?

c.

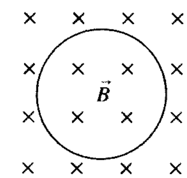
d. At which point or points is the flux changing least rapidly?

b and d.

30.4 Lenz's Law

30.5 Faraday's Law

10. Does the loop of wire have a clockwise current, a counterclockwise current, or no current under the following circumstances? Explain.



a. The magnetic field points into the page and its strength is increasing.

CCW. To oppose the change in flux (increasing into page), the $\vec{B}_{induced}$ must be out of the page. So, by the right-hand rule, the induced current must be CCW.

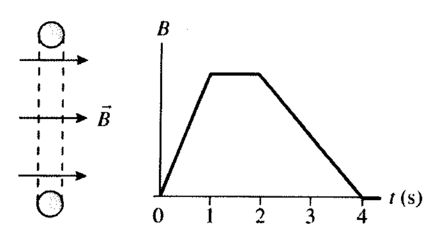
b. The magnetic field points into the page and its strength is constant.

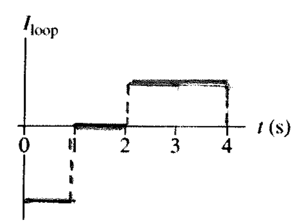
No current. There is no change in magnetic flux through the loop.

c. The magnetic field points into the page and its strength is decreasing.

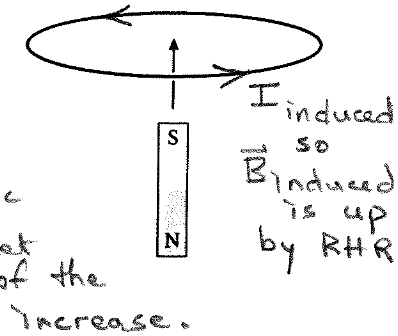
CW, since $\vec{B}_{induced}$ must be into the page to oppose the decreasing flux into the page. Current direction found by RHR.

11. A loop of wire is perpendicular to a magnetic field. The magnetic field strength as a function of time is given by the top graph. Draw a graph of the current in the loop as a function of time. Let a positive current represent a current that comes out of the top and enters the bottom. There are no numbers for the vertical axis, but your graph should have the correct shape and proportions.





12. A loop of wire is horizontal. A bar magnet is pushed toward the loop from below, along the axis of the loop.



a. What is the current direction in the loop as the magnet is approaching? Explain.

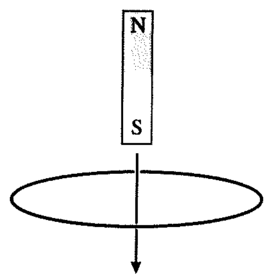
CCW as seen from above. The magnetic flux through the loop is increasing as $\vec{B}_{magnet}$ points down and increases. The direction of the current gives a $\vec{B}_{induced}$ that opposes this increase.

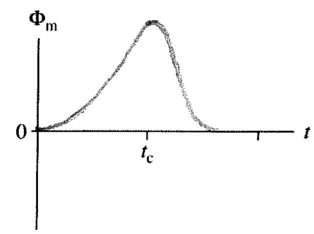
b. Is there a magnetic force on the loop? If so, in which direction? Explain. Hint: A current loop is a magnetic dipole.

Yes. The net magnetic force is up. The current loop is a magnetic dipole with its N pole up and S pole down. The loop experiences a repulsive force.

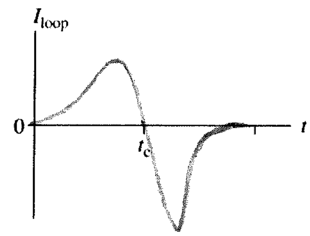
13. A bar magnet is dropped, south pole down, through the center of a loop of wire. The center of the magnet passes the plane of the loop at time t_c.

a. Sketch a graph of the magnetic flux through the loop as a function of time.



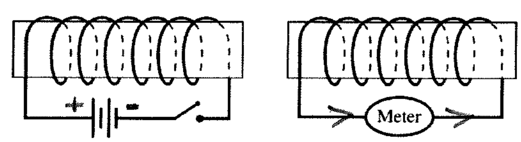


b. Sketch a graph of the current in the loop as a function of time. Let a clockwise current be a positive number and a counterclockwise current be a negative number.



Assume loop is held fixed and bar magnet is in free-fall. Graphs are not symmetric since flux changes more rapidly as falling speed increases.

14. a. Just after the switch on the left coil is closed, does current flow right to left or left to right through the current meter of the right coil? Or is the current zero? Explain.

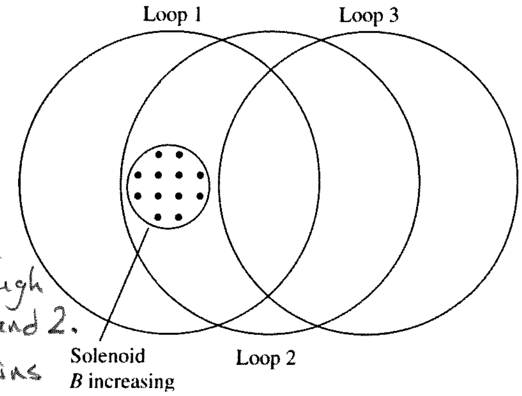


Just as the switch is closed, the left coil gives $\vec{B}$ pointing to the left, so this causes a rapid change in flux through the right coil. To oppose this change an induced $\vec{B}$ will be to the right. By RHR, the induced current giving this $\vec{B}$ must be $\boxed{\text{left to right}}$ through the meter.

b. Long after the switch on the left coil is closed, does current flow right to left or left to right through the current meter of the right coil? Or is the current zero? Explain.

The current is zero since $\vec{B}$ from the left coil and the flux through the right coil are not changing.

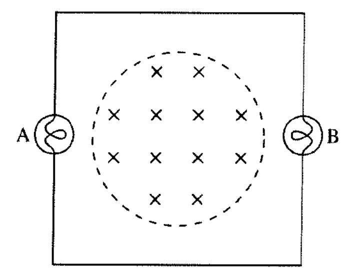
15. A solenoid is perpendicular to the page, and its field strength is increasing. Three circular wire loops of equal radii are shown. Rank in order, from largest to smallest, the size of the induced emf in the three rings.



Order: $\mathcal{E}_1 = \mathcal{E}_2 > \mathcal{E}_3$

Explanation:
Induced $\mathcal{E}$ depends on the changing flux through each loop. The changing flux through the solenoid is the same as that for loops 1 and 2. But for loop 3 the flux is zero and remains zero.

16. A conducting loop around a magnetic field contains two lightbulbs, A and B. The wires connecting the bulbs are ideal, with no resistance. The magnetic field is increasing rapidly.



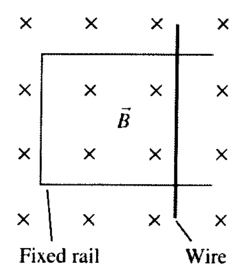
a. Do the bulbs glow? Why or why not?

Yes. The changing magnetic flux through the loop leads to a CCW induced current passing through the bulbs and the conducting loop.

b. If they glow, which bulb is brighter? Or are they equally bright? Explain.

Assuming identical bulbs they are equally bright since the same current flows through them.

17. A metal wire is resting on a U-shaped conducting rail. The rail is fixed in position, but the wire is free to move.



a. If the magnetic field is increasing in strength, does the wire:

i. Remain in place?	vi. Move out of the plane of the page, breaking contact with the rail?
ii. Move to the right?	
iii. Move to the left?	vii. Rotate clockwise?
iv. Move up on the page?	viii. Rotate clockwise?
v. Move down on the page?	ix. Some combination of these? If so, which?

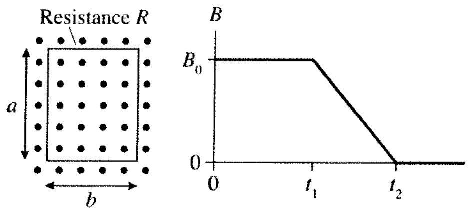
Explain your choice.

The wire moves to the left. The downward flux is increasing. To oppose this increase, an induced field must point up which requires a CCW induced current (i.e., up on the wire). Using $I\vec{\ell} \times \vec{B}$ gives a force that pulls the wire to the left.

b. If the magnetic field is decreasing in strength, which of the above happens? Explain.

The wire moves to the right. Now, the downward flux is decreasing so the induced current will be CW (i.e., down the wire). The force $I\vec{\ell} \times \vec{B}$ will pull the wire to the right.

18. The graph shows how the magnetic field changes through a rectangular loop of wire with resistance R. Draw a graph of the current in the loop as a function of time. Let a counterclockwise current be positive, a clockwise current be negative.
PSS 30.1



a. What is the magnetic flux through the loop at $t = 0$? $B_0(ab)$

b. Does this flux *change* between $t = 0$ and $t = t_1$? No.

c. Is there an induced current in the loop between $t = 0$ and $t = t_1$? No.

d. What is the magnetic flux through the loop at $t = t_2$? 0

e. What is the *change* in flux through the loop between t_1 and t_2? $\Delta\Phi = 0 - B_0(ab) = -B_0(ab)$

f. What is the time interval between t_1 and t_2? $t_2 - t_1$

g. What is the magnitude of the induced emf between t_1 and t_2? $\mathcal{E} = B_0(ab)/t_2 - t_1$

h. What is the magnitude of the induced current between t_1 and t_2? $\frac{B_0(ab)}{(t_2 - t_1)R}$

i. Does the magnetic field point out of or into the loop? Out of the loop.

j. Between t_1 and t_2, is the magnetic flux increasing or decreasing? Decreasing.

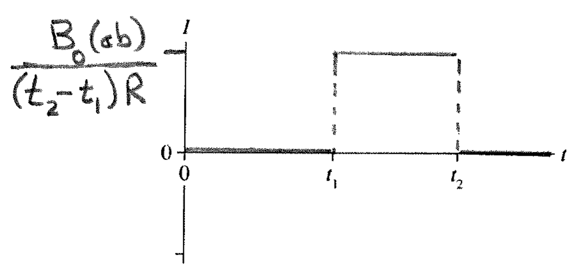
k. To oppose the *change* in the flux between t_1 and t_2, should the magnetic field of the induced current point out of or into the loop? Out of the loop.

l. Is the induced current between t_1 and t_2 positive or negative? Positive, CCW

m. Does the flux through the loop change after t_2? No.

n. Is there an induced current in the loop after t_2? No.

o. Use all this information to draw a graph of the induced current. Add appropriate labels on the vertical axis.



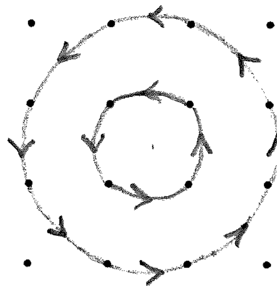
30.6 Induced Fields

19. Consider these two situations:

a. Draw the induced electric field.

$\vec{B}$-field rapidly increasing

b. Draw the induced electric field.



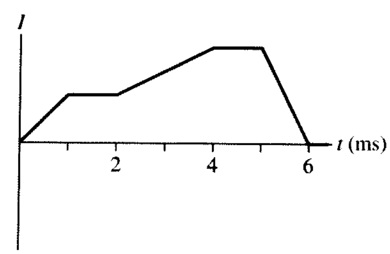
$\vec{B}$-field rapidly decreasing

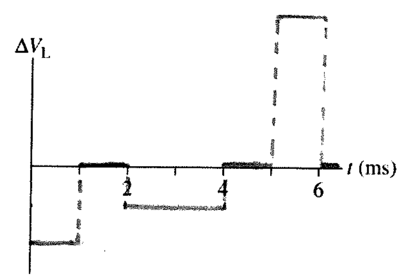
30.7 Induced Current: Three Applications

No exercises.

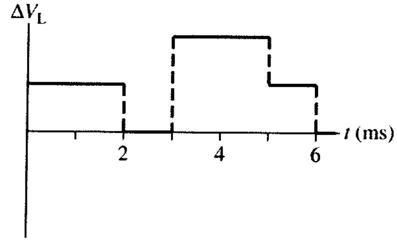
30.8 Inductors

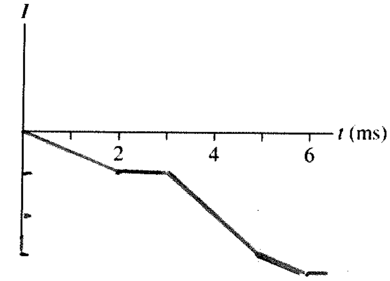
20. The figure shows the current through an inductor. Draw a graph showing the potential difference ΔV_L across the inductor. There are no numbers, but your graph should have the correct shape and proportions.





21. The figure shows the potential difference across an inductor. There is no current at $t = 0$. Draw a graph of the current through the inductor as a function of time. There are no numbers, but your graph should have the correct shape and proportions.





30.9 *LC* Circuits

22. An *LC* circuit oscillates at a frequency of 2000 Hz. What will the frequency be if the inductance is quadrupled?

Let $f_1 = \frac{\omega_1}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{L_1C}} = 2000\text{ Hz}$

Now $L_2 = 4L_1$ so $f_2 = \frac{\omega_2}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{L_2C}} = \frac{1}{2\pi}\sqrt{\frac{1}{4L_1C}} = \frac{f_1}{2} = \boxed{1000\text{Hz}}$

23. The capacitor in an *LC* circuit has maximum charge at $t = 1\ \mu s$. The current through the inductor next reaches a maximum at $t = 3\ \mu s$.

a. When will the inductor current reach a maximum in the opposite direction?

$t = 7\ \mu s$.

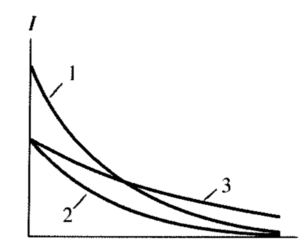
The capacitor charge and the inductor current are 90° out of phase.
So, $\Delta t = 2\mu s = T/4$. Must add $T/2 = 4\mu s$ to the time $t = 3s$.

b. What is the circuit's period of oscillation?

$T = 8\mu s$ since $\Delta t = 2\mu s = T/4$.

30.10 *LR* Circuits

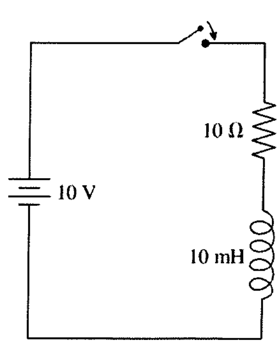
24. Three *LR* circuits are made with the same resistor but different inductors. The figure shows the inductor current as a function of time. Rank in order, from largest to smallest, the three inductances L_1, L_2, and L_3.



Order: $L_3 > L_2 > L_1$
Explanation:
$I = I_0 e^{-t/\tau}$ where $\tau = L/R$

As L increase, I decreases less rapidly.

25. a. What is the battery current immediately after the switch closes? Explain.



Zero. The inductor immediately oppose any current trying to pass through it. The battery current grows from zero to a steady state value.

b. What is the battery current after the switch has been closed a long time? Explain.

$I = \frac{\mathcal{E}}{R} = \frac{10V}{10\Omega} = 1A$

The current has reached its final steady state and $\frac{dI}{dt} = 0$.