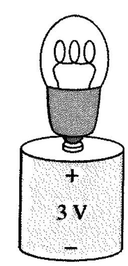
28 Fundamentals of Circuits

28.1 Circuit Elements and Diagrams

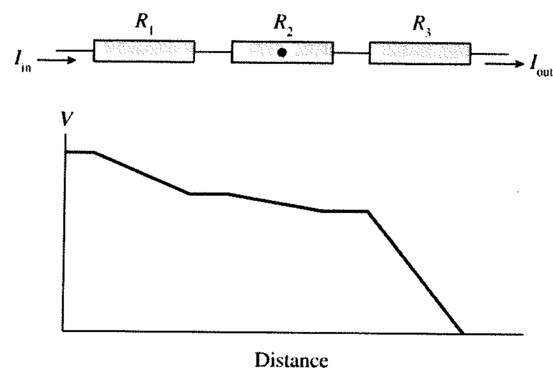
28.2 Kirchhoff's Laws and the Basic Circuit

1. The tip of a flashlight bulb is touching the top of a 3 V battery. Does the bulb light? Why or why not?



No, not a complete circuit. The bulb's metal tip and metal side which are insulated from each other are connected to opposite ends of the bulb's filament. To complete a circuit both of these metals must connect to the battery, and these metals must connect to opposite terminals of the battery.

2. Current I_{in} flows into three resistors connected together one after the other. The graph shows the value of the potential as a function of distance.



a. Is I_{out} greater than, less than, or equal to I_{in}? Explain.

Must be equal by conservation of current.

b. Rank in order, from largest to smallest, the three resistances R_1, R_2, and R_3.

Order: $R_3 > R_1 > R_2$

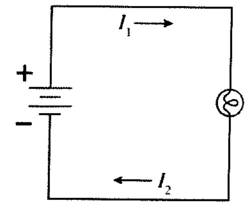
Explanation:

$R = \frac{\Delta V_R}{I}$. The current is the same, but the graph shows $\Delta V_{R_3} > \Delta V_{R_1} > \Delta V_{R_2}$

c. Is there an electric field at the point inside R_2 that is marked with a dot? If so, in which direction does it point? If not, why not?

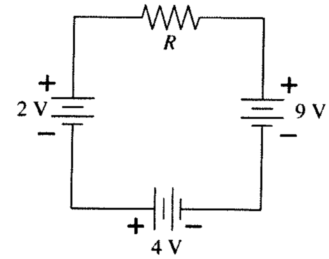
Yes, to the right. A resistor is just a conductor with a larger resistance than that of wires.

3. A flashlight bulb is connected to a battery and is glowing. Is current I_2 greater than, less than, or equal to current I_1? Explain.



Must be equal by conservation of current.

4. a. In which direction does current flow through resistor R?



Righ to left

b. Which end of R is more positive? Explain.

The right end. The current always enters the resistor on the high potential end.

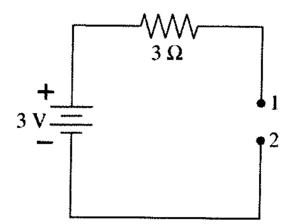
c. If this circuit were analyzed in a clockwise direction, what numerical value would you assign to ΔV_{res}? Why?

$\Delta V_{res} = +3\,V$. Positive since direction across the resistor is opposite the current. Kirchhoff's Loop Law gives $\Sigma(\Delta V)_i = +2V + \Delta V_{res} - 9V + 4V = 0$.

d. What value would ΔV_{res} have if the circuit were analyzed in a counterclockwise direction?

$\Delta V_{res} = -3V$ since the direction across the resistor is with the current.

5. The wire is broken on the right side of this circuit. What is the potential difference ΔV_{12} between points 1 and 2? Explain.



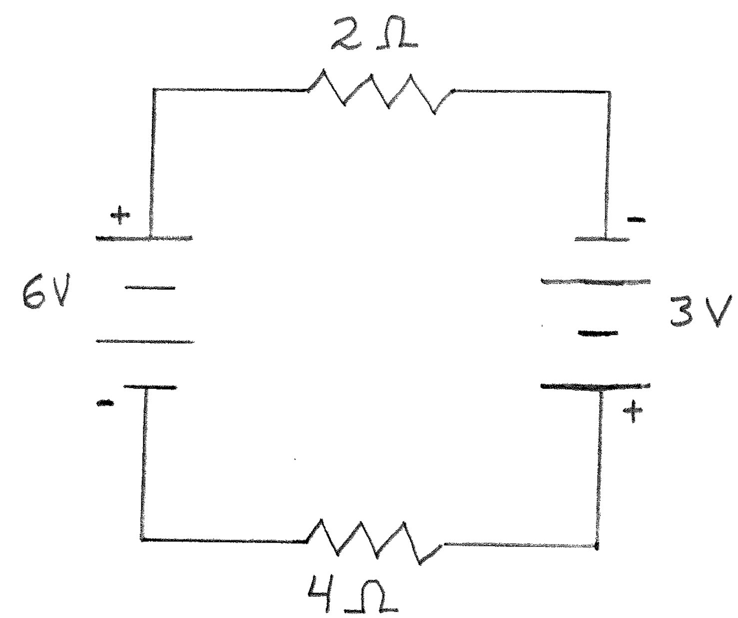
$\Delta V_{12} = 3V$

There is no current, so $\Delta V_R = 0$. Point 1 is at the same potential as the positive terminal of the battery, and point 2 same as negative terminal.

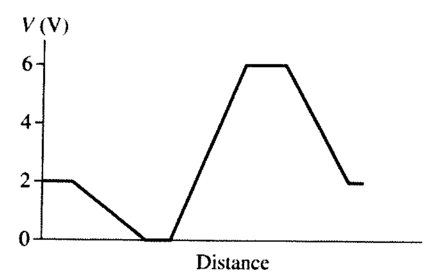
6. Draw a circuit for which the Kirchhoff's loop law equation is

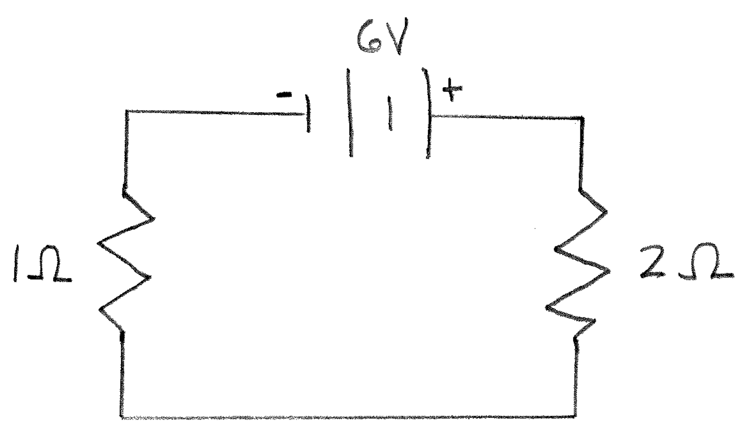
$$6\text{V} - I \cdot 2\Omega + 3\text{V} - I \cdot 4\Omega = 0$$

Assume that the analysis is done in a clockwise direction.



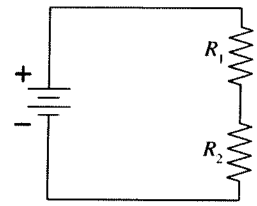
7. The current in a circuit is 2.0 A. The graph shows how the potential changes when going around the circuit in a clockwise direction, starting from the lower left corner. Draw the circuit diagram.





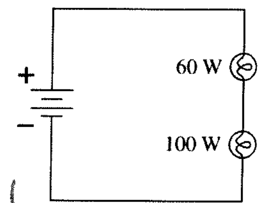
28.3 Energy and Power

8. This circuit has two resistors, with $R_1 > R_2$. Which of the two resistors dissipates the larger amount of power? Explain.



R_1 dissipates more power since the current is the same everywhere and $P = I^2R$ with $R_1 > R_2$.

9. A 60 W lightbulb and a 100 W lightbulb are placed one after the other in a circuit. The battery's emf is large enough that both bulbs are glowing. Which one glows more brightly? Explain.



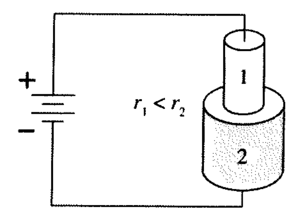
A bulb's wattage rating is the power a bulb will dissipate if operated with a potential difference of a standard household voltage (120V).

So, Wattage rating $= \dfrac{(120\text{V})^2}{R_{bulb}} \Rightarrow$ 60W bulb has higher R.

In the given circuit, the current is the same everywhere so $P = I^2R$ is greater for the 60W bulb since R is higher.

$\boxed{\text{The 60W bulb glows more brightly.}}$

10. Two conductors of equal lengths are connected to a battery by ideal wires. The conductors are made of the same material but have different radii r. Which of the two conductors dissipates the larger amount of power? Explain.

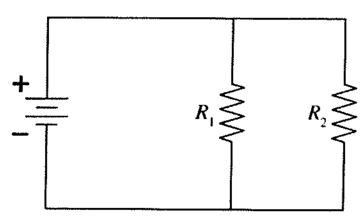


Conductor 1.

$P = I^2R$, same I but $R_1 > R_2$ since

$R = \dfrac{\rho L}{A}$ and $\rho_1 = \rho_2$, $L_1 = L_2$ but $A_1 < A_2$

11. The circuit shown has a battery and two resistors, with $R_1 > R_2$. Which of the two resistors dissipates the larger amount of power? Explain your reasoning.



R_2 dissipates more power.

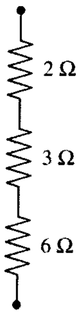
$P = \dfrac{(\Delta V)^2}{R}$ and both resistors have the same potential drop because they are connected in parallel.

28.4 Series Resistors

28.5 Real Batteries

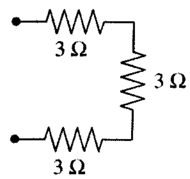
12. What is the equivalent resistance of each group of resistors?

a.



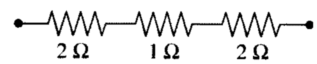
$R_{eq} =$ 11 Ω

b.



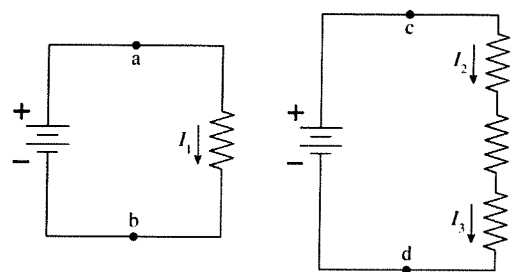
$R_{eq} =$ 9 Ω

c.



$R_{eq} =$ 5 Ω

13. The figure shows two circuits. The two ideal batteries are identical and the four resistors all have exactly the same resistance.



a. Is ΔV_{ab} larger than, smaller than, or equal to ΔV_{cd}? Explain.

$\Delta V_{ab} = \Delta V_{cd}$

By Loop Law: $\Delta V_{ab} + \Delta V_{bat} = 0$. Likewise $\Delta V_{cd} + \Delta V_{bat} = 0$.
Identical batteries so same ΔV_{bat} implies $\Delta V_{ab} = \Delta V_{cd}$.

b. Rank in order, from largest to smallest, the currents I_1, I_2, and I_3.

Order: $I_1 > I_2 = I_3$

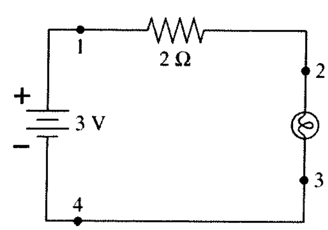
Explanation:

Let $\Delta V = \Delta V_{ab} = \Delta V_{cd}$

$I_1 = \frac{\Delta V}{R}$ but $I_2 = I_3 = \frac{\Delta V}{3R}$

same current by conservation of current.

14. The lightbulb in this circuit has a resistance of 1 Ω.



a. What are the values of:

ΔV_{12} −2V

ΔV_{23} −1V

ΔV_{34} 0V

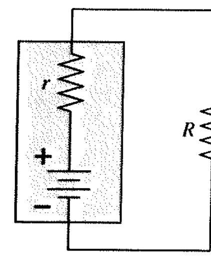
b. Suppose the bulb is now removed from its socket. Then what are the values of:

ΔV_{12} 0V

ΔV_{23} −3V

ΔV_{34} 0V

15. If the value of R is increased, does ΔV_{bat} increase, decrease, or stay the same? Explain.

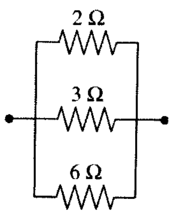


ΔV_{bat} will increase since $\Delta V_{bat} = \frac{R}{R+r}\mathcal{E}$

28.6 Parallel Resistors

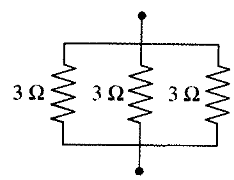
16. What is the equivalent resistance of each group of resistors?

a.



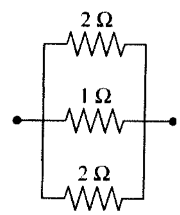
$R_{eq} =$ 1 Ω

b.



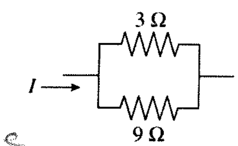
$R_{eq} =$ 1 Ω

c.



$R_{eq} =$ 0.5 Ω

17. a. What fraction of current I goes through the 3 Ω resistor?

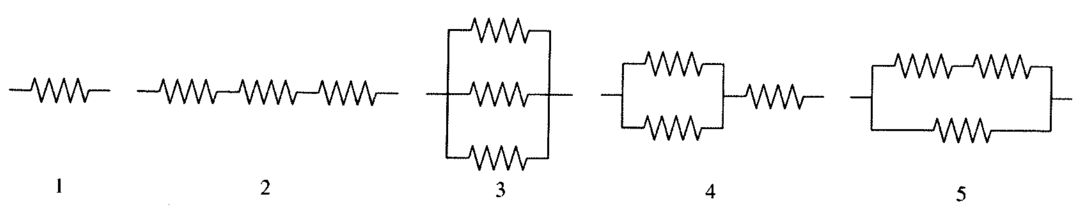


$\frac{3}{4}I$ must pass through the 3 Ω resistor while $\frac{1}{4}I$ passes through the 9 Ω one. This is required to have the same $\Delta V_R = IR$ across both resistors.

b. If the 9 Ω resistor is replaced with a larger resistor, will the fraction of current going through the 3 Ω resistor increase, decrease, or stay the same?

Increases current through 3 Ω resistor.
Current will favor the path of least resistance.

18. The figure shows five combinations of identical resistors. Rank in order, from largest to smallest, the equivalent resistances $(R_{eq})_1$ to $(R_{eq})_5$.



Order: $(R_{eq})_2 > (R_{eq})_4 > (R_{eq})_1 > (R_{eq})_5 > (R_{eq})_3$

Explanation:

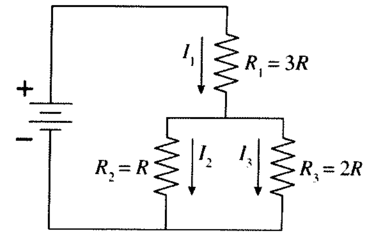
$(R_{eq})_1 = R$ $\quad (R_{eq})_2 = R+R+R = 3R$ $\quad (R_{eq})_3 = \frac{1}{\frac{1}{R}+\frac{1}{R}+\frac{1}{R}} = \frac{R}{3}$

$(R_{eq})_4 = \frac{1}{\frac{1}{R}+\frac{1}{R}} + R = \frac{3}{2}R$ $\quad (R_{eq})_5 = \frac{1}{\frac{1}{2R}+\frac{1}{R}} = \frac{2R}{3}$

28.7 Resistor Circuits

28.8 Getting Grounded

19. Rank in order, from largest to smallest, the three currents I_1 to I_3.

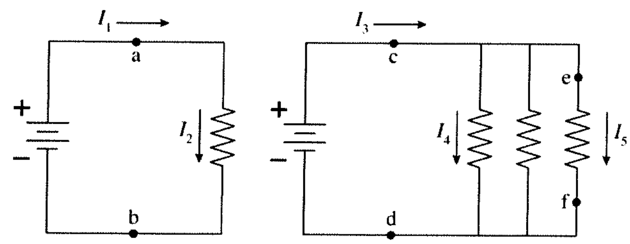


Order: $I_1 > I_2 > I_3$

Explanation:

By Kirchhoff's Junction Rule $I_1 = I_2 + I_3$. Also $I_2 > I_3$ since $R_2 < R_3$ and the potential drop across R_2 and R_3 is the same since they are connected in parallel. $I = \frac{\Delta V}{R}$.

20. The two ideal batteries are identical and the four resistors all have exactly the same resistance.



a. Compare ΔV_{ab}, ΔV_{cd}, and ΔV_{ef}. Are they all the same? If not, rank them in decreasing order. Explain your reasoning.

They are all the same. Points a, c and e are all at the same potential as the positive electrode on the battery. Points b, d and f are all the same potential as the negative electrode.

b. Rank in order, from largest to smallest, the five currents I_1 to I_5.

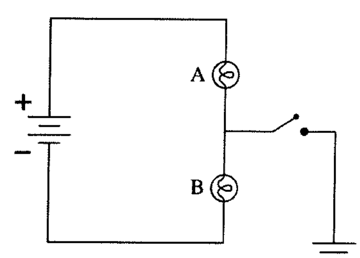
Order: $I_3 > I_1 = I_2 = I_4 = I_5$

Explanation:

By conservation of current $I_1 = I_2$. Also $I_1 = \frac{\varepsilon}{R}$

$$I_3 = \frac{\varepsilon}{R/3} = 3I_1 \qquad I_4 = \frac{\varepsilon}{R} = I_5$$

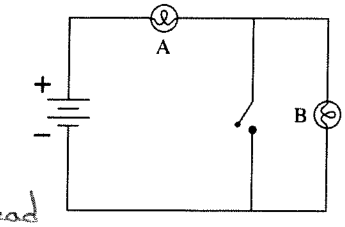
21. Bulbs A and B are identical and initially both are glowing. Then the switch is closed. What happens to each bulb? Does its brightness increase, stay the same, decrease, or go out? Explain.



Stays the same. The current and potential difference across each bulb is unchanged. The ground wire sets the location of V = 0V but no current flows through it since it is not part of a complete circuit.

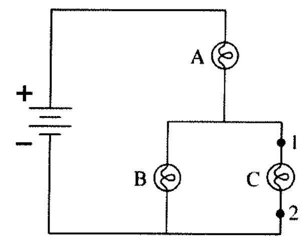
Exercises 22–25: Assume that all wires are ideal (zero resistance) and that all batteries are ideal (constant potential difference).

22. Initially bulbs A and B are glowing. Then the switch is closed. What happens to each bulb? Does it get brighter, stay the same, get dimmer, or go out? Explain your reasoning.



Bulb A glows brighter, and bulb B goes out. The current does not enter bulb B but instead follows the path with the closed switch since this path has essentially zero resistance. Without the resistance from bulb B, the current through bulb A increases, so it glows brighter ($P=I^2R$).

23. a. Bulbs A, B, and C are identical. Rank in order, from most to least, the brightnesses of the three bulbs.



Order: A > B = C

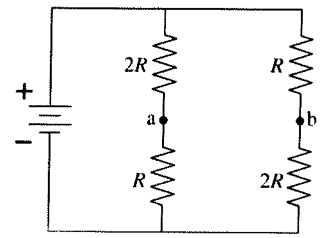
Explanation:

All current flowing from the battery must go through bulb A, but then the current splits evenly between bulb B and bulb C. With the same resistance for each bulb, the larger current means greater brightness, $P=I^2R$.

b. Suppose a wire is connected between points 1 and 2. What happens to each bulb? Does it get brighter, stay the same, get dimmer, or go out? Explain.

Bulbs B and C go out, but bulb A gets brighter. The current follows the path of the added wire since it has essentially zero resistance, so no current enters bulbs B or C. Without resistance from bulbs B and C, the current through bulb A increases, so it shines brighter, $P=I^2R$.

24. a. Consider the points a and b. Is the potential difference $\Delta V_{ab} = 0$? If so, why? If not, which point is more positive?

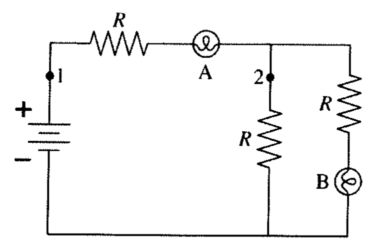


No, because the voltage drop across the 2R resistor is greater than the voltage drop across the R resistor. $\Delta V = IR$ and $I_a = I_b$. Point b is more positive.

b. If a wire is connected between points a and b, does a current flow through it? If so, in which direction—to the right or to the left? Explain.

To the left from b to a. The connecting wire will cause a and b to be at equal potentials. However, the current will favor a path of least resistance which is through the top R resistor, then from b to a; then through the bottom R resistor. (The current that passes through each 2R resistor is half that which passes through each R resistor.)

25. Bulbs A and B are identical. Initially both are glowing.



a. Bulb A is removed from its socket. What happens to bulb B? Does it get brighter, stay the same, get dimmer, or go out? Explain.

Bulb B goes out. With bulb A out of its socket there is no closed path so there is no current.

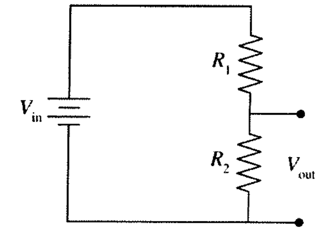
b. Bulb A is replaced. Bulb B is then removed from its socket. What happens to bulb A? Does it get brighter, stay the same, get dimmer, or go out? Explain.

Bulb A gets dimmer: The current in the circuit decreases because the equivalent resistance increases when no current enters bulb B.

c. The circuit is restored to its initial condition. A wire is then connected between points 1 and 2. What happens to the brightness of each bulb?

Bulb A goes out because the current follows the path of zero resistance from 1 to 2 and does not enter bulb A. Bulb B gets brighter since more current passes through it now that the resistance of the circuit is less.

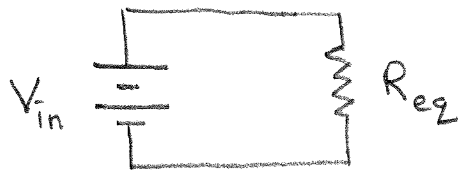
26. Real circuits frequently have a need to reduce a voltage to a smaller value. This is done with a *voltage-divider circuit* such as the one shown here. We've shown the input voltage V_{in} as a battery, but in practice, it might be a voltage signal produced by some other circuit, such as the receiver circuit in your television.
PSS 28.1



a. Is this a series or a parallel circuit? Series

b. Symbolically, what is the value of the equivalent resistance? $R_{eq} =$ $R_1 + R_2$

c. Redraw the circuit with the two resistors replaced with the equivalent resistance.



d. You now have one resistor connected directly across the battery. Find the voltage across and the current through this resistor. Write your answers in terms of V_{in}, R_1, and R_2.

$$\Delta V_{R_{eq}} = V_{in}, \qquad I = \frac{V_{in}}{R_{eq}} = \frac{V_{in}}{R_1 + R_2}$$

$\Delta V =$ V_{in} $\qquad I =$ $V_{in}/(R_1 + R_2)$

e. Which of these, ΔV or I, is the same for R_1 and R_2 as for R_{eq}? I

f. Using Ohm's law and your answer to e, what is V_{out}? It's most useful to write your answer as $V_{out} =$ something $\times V_{in}$.

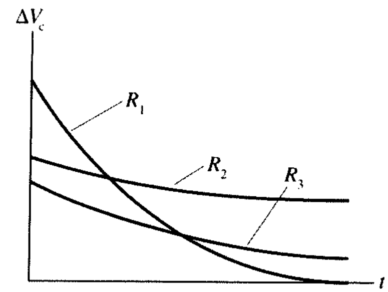
$$V_{out} = IR_2 = \left(\frac{V_{in}}{R_1 + R_2}\right) R_2$$

$V_{out} =$ $\left(\frac{R_2}{R_1 + R_2}\right) V_{in}$

g. What is V_{out} if $R_2 = 2R_1$? $V_{out} =$ $(2/3)\ V_{in}$

28.9 *RC* Circuits

27. The graph shows the voltage as a function of time on a capacitor as it is discharged (separately) through three different resistors. Rank in order, from largest to smallest, the values of the resistances R_1 to R_3.

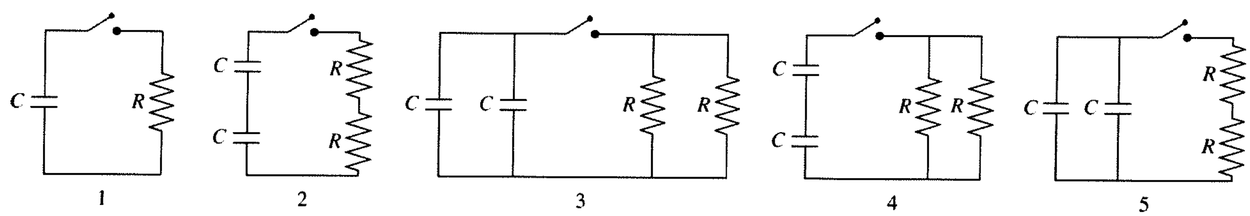


Order: $R_2 > R_3 > R_1$

Explanation:

$\tau = RC$. The decay time is longest for the R_2 curve and shortest for the R_1 curve.

28. The capacitors in each circuit are fully charged before the switch closes. Rank in order, from largest to smallest, the time constants τ_1 to τ_5 with which the capacitors will discharge after the switch closes.



Order: $\tau_5 > \tau_1 = \tau_2 = \tau_3 > \tau_4$

Explanation:

$\tau_1 = RC$

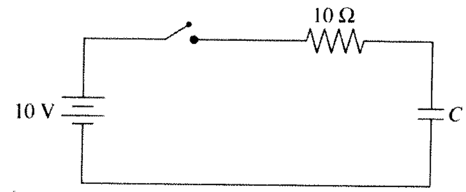
$\tau_2 = (2R)(\frac{C}{2}) = \tau_1$

$\tau_3 = (\frac{R}{2})(2C) = \tau_1$

$\tau_4 = (\frac{R}{2})(\frac{C}{2}) = \frac{1}{4}\tau_1$

$\tau_5 = (2R)(2C) = 4\tau_1$

29. The charge on the capacitor is zero when the switch closes at $t = 0$ s.



a. What will be the current in the circuit after the switch has been closed for a long time? Explain.

After the switch has been closed a long time the capacitor will be fully charged and the current will approach zero.

b. Immediately after the switch closes, before the capacitor has had time to charge, the potential difference across the capacitor is zero. What must be the potential difference across the resistor in order to satisfy Kirchhoff's loop law? Explain.

-10 V.

Apply Kirchhoff's loop law going clockwise

$$0 = \Delta V_{bat} + \Delta V_{res} + \overset{0}{\cancel{\Delta V_C}} = \mathcal{E} + \Delta V_{res}$$

so

$$\Delta V_{res} = -\mathcal{E} = -10\text{V}. \quad \text{Magnitude } \Delta V_R = 10\text{V}.$$

c. Based on your answer to part b, what is the current in the circuit immediately after the switch closes?

$$I = \frac{\Delta V_R}{R} = \frac{10\text{V}}{10\,\Omega} = 1\text{A}$$

d. Sketch a graph of current versus time, starting from just before $t = 0$ s and continuing until the switch has been closed a long time. There are no numerical values for the horizontal axis, so you should think about the *shape* of the graph.