27 Current and Resistance

27.1 The Electron Current

1. A lightbulb is connected with wires to a battery, and the bulb is glowing. Are simple observations and measurements you can make on this circuit able to distinguish a current composed of positive charge carriers from a current composed of negative charge carriers? If so, describe how you can tell which it is. If not, why not?

No. A flow of positive charges in one direction would be the same as a flow of negative charges in the opposite direction. Either could explain the simple observations and measurements you can make.

2. One model of current consists of the motion of discrete, charged particles. Another model is that current is the flow of a continuous charged fluid. Do simple experiments provide evidence in favor of either one of these models? Explain.

No. For simple experiments with wires, bulbs, batteries, etc., the density of flowing discrete, charged particles (electrons) is so high that a continuous charged fluid model can give the same simple observations and measurements as that of a discrete, charged particles model.

3. Are the charge carriers always electrons? If so, why is this the case? If not, describe a situation in which a current is due to some other charge carrier.

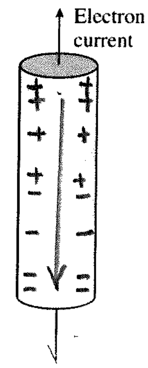
No. Electrons are the charge carriers in metals. However, other conductors, like ionic solutions or semiconductors, have charge carriers that are not electrons.

27.2 Creating a Current

4. The electron drift speed in a wire is exceedingly slow—typically only a fraction of a millimeter per second. Yet when you turn on a light switch, a light bulb several meters away seems to come on instantly. Explain how to resolve this apparent paradox.

The entire length of the wire and the light bulb filament are already full of conduction electrons before turning on the switch. When you flip the switch, an electric field instantly pushes all of these electrons. Those electrons at the very end of the wire connected to the bulb immediately move ("drift") into the bulb.

5. The figure shows a segment of a current-carrying metal wire.



a. Is there an electric field inside the wire? If so, draw and label an arrow on the figure to show its direction. If not, why not?

Yes. The field is drawn opposite the electron current.

b. If there is an electric field, draw on the figure a possible arrangement of charges that could be the source charges causing the field. Nonuniform surface charge density is shown

6. a. If the electrons in a current-carrying wire collide with the positive ions *more* frequently, does their drift speed increase or decrease? Explain.

If the collisions occur more frequently then the mean time between collisions, τ, decreases and the net directional motion slows down (i.e., slower drift speed, v_d).
$v_d = \frac{e\tau}{m}E$. Their drift speed decreases.

b. Does an increase in the collision frequency make the wire a better conductor or a worse conductor? Explain.

A worse (or weaker) conductor.
$i_e = n_e A v_d$
A slower v_d (drift speed) means a weaker current, so fewer electrons passing through the wire each second.

c. Would you expect a metal to be a better conductor at high temperature or at low temperature? Explain.

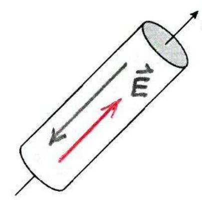
Metals are better conductors at low temperatures.
An increase in random thermal vibrations of lattice atoms at higher temperatures causes more electron collisions, slower drift velocity and thus weaker currents (poor conductivity).

27.3 Current and Current Density

7. What is the difference between current and current density?

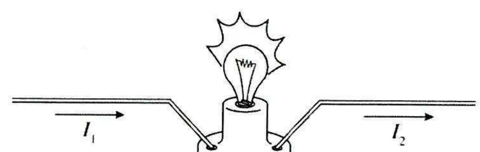
Current is the rate at which charges move through a wire. Current density, J, is the current per square meter of cross section. $J = \frac{I}{A}$.

8. The figure shows a segment of a current-carrying metal wire.
   a. Draw an arrow on the figure, using a **black** pen or pencil, to show the direction of motion of the charge carriers.
   b. Draw an arrow on the figure, using a **red** pen or pencil, to show the direction of the electric field.

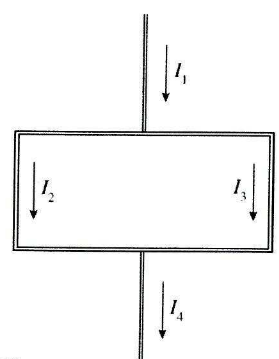


9. Is I_2 greater than, less than, or equal to I_1? Explain.

$I_2 = I_1$ by the law of conservation of current.
The current entering the bulb must equal the current leaving the bulb.



10. All wires in this figure are made of the same material and have the same diameter. Rank in order, from largest to smallest, the currents I_1 to I_4.

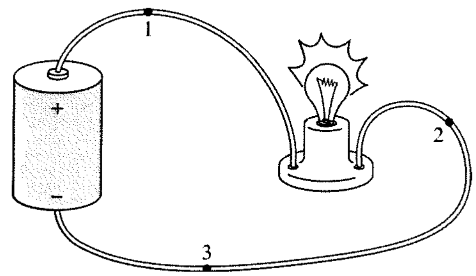


Order: $I_1 = I_4 > I_2 = I_3$

Explanation:

Current must be conserved.
The sum of the current into a junction must equal the sum of the current leaving, so
$I_1 = I_2 + I_3$ and $I_4 = I_2 + I_3$. Thus, $I_1 = I_4$.
The same material and same diameter requires $I_2 = I_3$ (an even split).

11. A lightbulb is connected to a battery with 1-mm-diameter wires. The bulb is glowing.



a. Draw arrows at points 1, 2, and 3 to show the direction of the electric field at those points. (The points are *inside* the wire.)

b. Rank in order, from largest to smallest, the field strengths E_1, E_2, and E_3.

Order: $E_1 = E_2 = E_3$

Explanation:

The current is the same at every point in the wire and $E \propto I$.

12. A wire carries a 4 A current. What is the current in a second wire that delivers twice as much charge in half the time?

$$I_1 = \frac{\Delta Q_1}{\Delta t_1} = 4\,\text{A}$$

$$I_2 = \frac{\Delta Q_2}{\Delta t_2} = \frac{2\Delta Q_1}{\frac{1}{2}\Delta t_1} = 4\frac{\Delta Q_1}{\Delta t_1} = 4(4\,\text{A}) = \boxed{16\,\text{A}}$$

13. The current density in a wire is 1000 A/m^2. What will the current density be if the current is doubled and the wire's diameter is halved?

$J_1 = \frac{I_1}{A_1}$ where circular cross-sectional area $A_1 \propto d_1^2$

$J_1 = 1000\ \text{A/m}^2$ as given

$d_2 = d_1/2$ so $A_2 = \left(\frac{1}{2}\right)^2 A_1 = \frac{1}{4}A_1$

$I_2 = 2I_1$

$$J_2 = \frac{I_2}{A_2} = \frac{2I_1}{A_1/4} = 8\frac{I_1}{A_1} = 8(1000\ \text{A/m}^2) = 8000\ \text{A/m}^2$$

27.4 Conductivity and Resistivity

14. Metal 1 and metal 2 are each formed into 1-mm-diameter wires. The electric field needed to cause a 1 A current in metal 1 is larger than the electric field needed to cause a 1 A current in metal 2. Which metal has the larger conductivity? Explain.

Metal 2. A metal is a good conductor with large conductivity if only a weak electric field is needed to produce electron flow (current). If a larger field is needed to produce the same current then the electrons are encountering more resistance to flow and the metal has smaller conductivity.

15. If a metal is heated, does its conductivity increase, decrease, or stay the same? Explain.

Conductivity decreases. An increase in thermal vibrations of lattice atoms at higher temperatures causes more electron collisions, slower drift velocity and weaker currents (poor conductivity).

16. Wire 1 and wire 2 are made from the same metal. Wire 1 has twice the diameter and half the electric field of wire 2. What is the ratio I_1/I_2?

$$\frac{I_1}{I_2} = \frac{E_1 \sigma A_1}{E_2 \sigma A_2}$$ same metal so same σ divides out

$$\frac{I_1}{I_2} = \frac{E_1 A_1}{E_2 A_2} = \frac{E_1 \pi (d_1/2)^2}{E_2 \pi (d_2/2)^2} = \frac{E_1 d_1^2}{E_2 d_2^2} = \frac{(E_2/2)(2d_2)^2}{E_2 d_2^2} = \boxed{2}$$

17. Wire 1 and wire 2 are made from the same metal. Wire 2 has a larger diameter than wire 1. The electric field strengths E_1 and E_2 in the wires are equal.

a. Compare the values of the two current densities. Is J_1 greater than, less than, or equal to J_2? Explain.

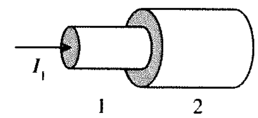
Given same metal so same σ. Also same E.
$J = \sigma E$ so $J_1 = J_2$.

b. Compare the values of the currents I_1 and I_2.

$I_1 = \sigma_1 E_1 A_1$ and $I_2 = \sigma_2 E_2 A_2$
Given $\sigma_1 = \sigma_2$, $E_1 = E_2$ but $A_1 < A_2$
so $I_1 < I_2$

c. Compare the values of the electron drift speeds $(v_d)_1$ and $(v_d)_2$.

$J = n_e e v_d$. Same J.
Same number density of electrons since same metal.
So, v_d same, $(v_d)_1 = (v_d)_2$



18. A wire consists of two segments of different diameters but made from the same metal. The current in segment 1 is I_1.

a. Compare the values of the currents in the two segments. Is I_2 greater than, less than, or equal to I_1? Explain.

$I_1 = I_2$. Due to conservation of current, the current everywhere in the wire must be the same. Current leaving segment 1 must be same as current entering segment 2.

b. Compare the values of the current densities J_1 and J_2.

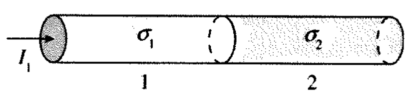
Drawing reveals cross-sectional areas, $A_1 < A_2$
so $J_1 = \frac{I}{A_1}$ and $J_2 = \frac{I}{A_2}$ gives $\boxed{J_1 > J_2}$

c. Compare the strengths of the electric fields E_1 and E_2 in the two segments.

Same metal so same σ throughout wire.
$E_1 = \frac{J_1}{\sigma}$ and $E_2 = \frac{J_2}{\sigma}$ gives $\boxed{E_1 > E_2}$.

d. Compare the values of the electron drift speeds $(v_d)_1$ and $(v_d)_2$.

Same metal so same electron number density n_e throughout wire.
$J = nev_d$ and $J_1 > J_2$ gives $\boxed{(v_d)_1 > (v_d)_2}$



19. A wire consists of two equal-diameter segments. Their conductivities differ, with $\sigma_2 > \sigma_1$. The current in segment 1 is I_1.

a. Compare the values of the currents in the two segments. Is I_2 greater than, less than, or equal to I_1? Explain.

Due to conservation of current, the current everywhere in the wire must be the same.
$\boxed{I_2 = I_1}$

b. Compare the strengths of the current densities J_1 and J_2.

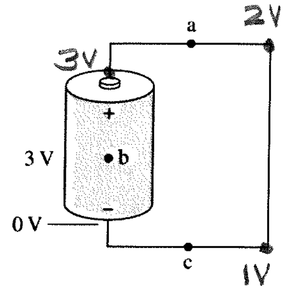
Since the cross-sectional area A does not change, we have
$J_1 = \frac{I_1}{A}$ and $J_2 = \frac{I_2}{A}$ so $\boxed{J_1 = J_2}$

c. Compare the strengths of the electric fields E_1 and E_2 in the two segments.

Let $J = J_1 = J_2$ then the values
$E_1 = \frac{J}{\sigma_1}$ and $E_2 = \frac{J}{\sigma_2}$ with $\sigma_2 > \sigma_1$ imply $\boxed{E_1 > E_2}$

27.5 Resistance and Ohm's Law

20. A continuous metal wire connects the two ends of a 3 V battery with a rectangular loop. The negative terminal of the battery has been chosen as the point where $V = 0$ V.



   a. Locate and label the approximate points along the wire where $V = 3$ V, $V = 2$ V, and $V = 1$ V.

   b. Points a and c are *inside* the wire. Point b is inside the battery. Does the electric field at a, b, and c point left, right, up, or down? Or is $\vec{E} = \vec{0}$?

   $\vec{E}_a$ Right $\vec{E}_b$ Down $\vec{E}_c$ Left

   c. In moving through the *wire* from the negative to the positive battery terminal, does the electric potential increase, decrease, or not change? If the potential changes, by how much does it change?

   The potential increases from 0 V to 3V.
   The potential change is 3V. $\Delta V = 3V$.

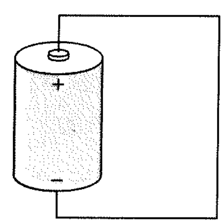
   d. In moving through the *battery* from the negative to the positive battery terminal, does the potential increase, decrease, or not change? If the potential changes, by how much does it change?

   The potential increases from 0V to 3V.
   The potential change is 3V. $\Delta V = 3V$.

   e. In moving all the way around the loop in a clockwise direction, is the net change in the potential positive, negative, or zero?

   Zero. Start at the negative terminal, then move through the battery to gain 3V; then move from positive to negative terminal to lose 3V. $\Delta V_{\text{entire loop}} = 0$.

21. a. Which direction—clockwise or counterclockwise—does an electron travel through the wire? Explain.



   Counterclockwise. The electron will be repelled by the negative terminal and attracted to the positive terminal.

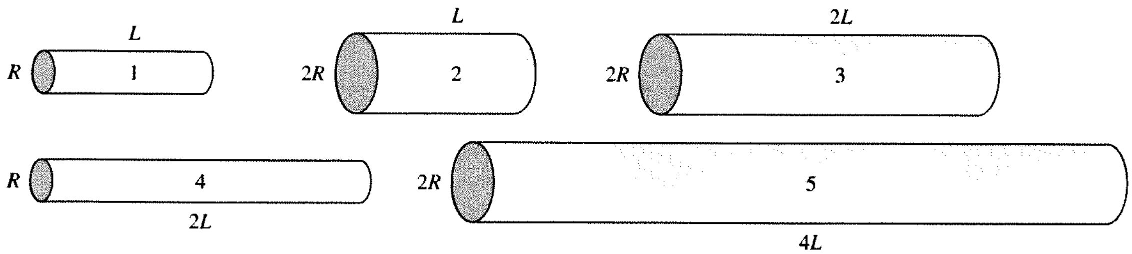
   b. Does an electron's electric potential energy increase, decrease, or stay the same as it moves through the wire? Explain.

   Decreases. A charge always moves in the direction of decreasing potential energy as it gains kinetic energy.

   c. If you answered "decrease" in part b, where does the energy go? If you answered "increase" in part b, where does the energy come from?

   The potential energy is transformed into thermal energy in the wire.

22. The wires below are all made of the same material. Rank in order, from largest to smallest, the resistances R_1 to R_5 of these wires.



Order: $R_4 > R_1 = R_5 > R_3 > R_2$

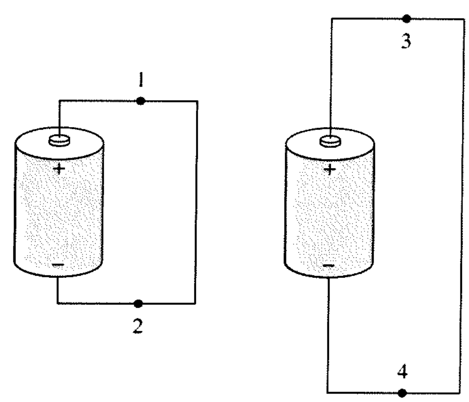
Explanation:

Resistance $R = \frac{\rho L}{A}$ same material, same ρ. $A \propto (\text{radius})^2$

$R_1 = \frac{\rho L}{\pi (R)^2}$, $R_2 = \frac{\rho L}{\pi (2R)^2} = \frac{1}{4} R_1$, $R_3 = \frac{\rho (2L)}{\pi (2R)^2} = \frac{1}{2} R_1$

$R_4 = \frac{\rho (2L)}{\pi R^2} = 2R_1$, $R_5 = \frac{\rho (4L)}{\pi (2R)^2} = R_1$

23. The two circuits use identical batteries and wires of equal diameters. Rank in order, from largest to smallest, the currents I_1, I_2, I_3, and I_4 at points 1 to 4.



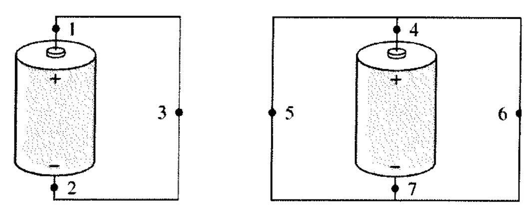
Order: $I_1 = I_2 > I_3 = I_4$

Explanation:

Conservation of current requires $I_1 = I_2$ and $I_3 = I_4$

However, longer wire gives more resistance, so I_3 and I_4 are weaker.

24. The two circuits use identical batteries and wires of equal diameters. Rank in order, from largest to smallest, the currents I_1 to I_7 at points 1 to 7.



Order: $I_4 = I_7 > I_1 = I_2 = I_3 = I_5 = I_6$

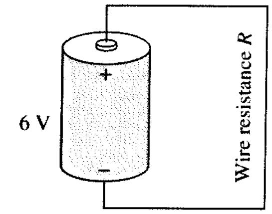
Explanation:

Conservation of current gives $I_1 = I_2 = I_3$

Junction: $\Sigma I_{in} = \Sigma I_{out}$ gives $I_4 = I_5 + I_6 = I_7$

Identical batteries and wires gives $I_3 = I_5 = I_6$

25. A wire is connected to the terminals of a 6 V battery. What is the potential difference ΔV_{wire} between the two ends of the wire, and what is the current I through the wire, if the wire has the following resistances:



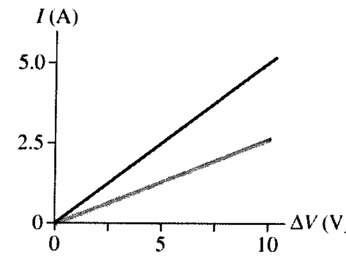
a. $R = 1\ \Omega$ $\quad \Delta V_{\text{wire}} =$ 6V $\quad I =$ 6A

b. $R = 2\ \Omega$ $\quad \Delta V_{\text{wire}} =$ 6V $\quad I =$ 3 A

c. $R = 3\ \Omega$ $\quad \Delta V_{\text{wire}} =$ 6 V $\quad I =$ 2 A

d. $R = 6\ \Omega$ $\quad \Delta V_{\text{wire}} =$ 6 V $\quad I =$ 1 A

26. The graph shows the current-versus-potential-difference relationship for a resistor R.

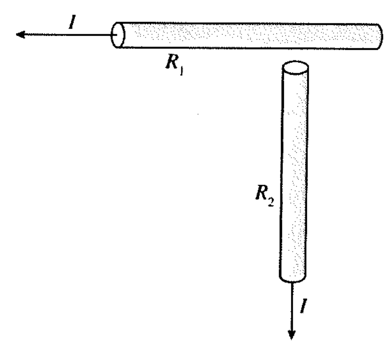


a. What is the numerical value of R?

$$R = \frac{1}{\text{slope}} = \frac{10\text{V}}{5\text{A}} = 2\,\Omega$$

b. Suppose the length of the resistor is doubled. On the figure, draw the current-versus-potential-difference graph for the longer resistor.

27. For resistors R_1 to R_2:



a. Which end (left, right, top, or bottom) is more positive?

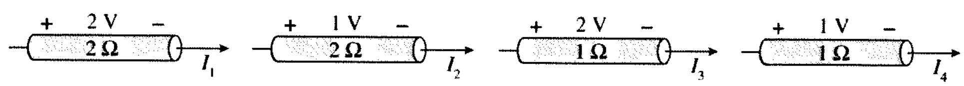
R_1: right $\quad R_2$: top

b. In which direction (such as left to right or top to bottom) does the potential decrease?

R_1: right to left

R_2: top to bottom

28. Rank in order, from largest to smallest, the currents I_1 to I_4 through these four resistors.



Order: $I_3 > I_1 = I_4 > I_2$

Explanation:

$$I = \frac{\Delta V}{R}$$

$$I_1 = \frac{2\text{V}}{2\,\Omega} = 1\text{A} \qquad I_2 = \frac{1\text{V}}{2\,\Omega} = 0.5\text{A}$$

$$I_3 = \frac{2\text{V}}{1\,\Omega} = 2\text{A} \qquad I_4 = \frac{1\text{V}}{1\,\Omega} = 1\text{A}$$

29. Which, if any, of these statements are true? (More than one may be true.)

i. A battery supplies the energy to a circuit.
ii. A battery is a source of potential difference. The potential difference between the terminals of the battery is always the same.
iii. A battery is a source of current. The current leaving the battery is always the same.

Explain your choice or choices.

i. True. The chemical reactions in the electrolytes separate the positive and negative charges. This creates a potential difference. The charges flowing in the circuit have energy due to this potential difference.

ii True only for ideal batteries. Real batteries lose energy over time so the potential difference between the terminals is not always the same.

iii False. The current leaving the battery depends on the resistance in the circuit.