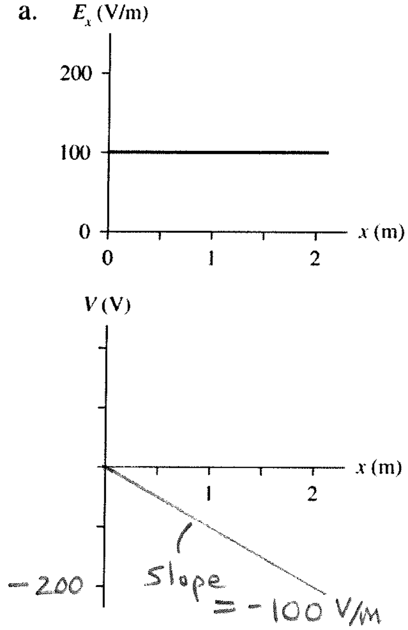
26 Potential and Field

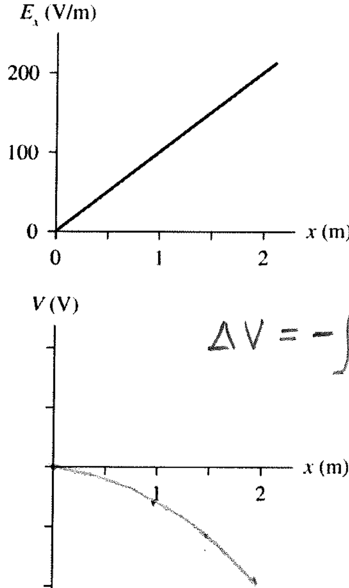
26.1 Connecting Potential and Field

1. The top graph shows the x-component of $\vec{E}$ as a function of x. On the axes below the graph, draw the graph of V versus x in this same region of space. Let $V = 0$ V at $x = 0$ m. Include an appropriate vertical scale. (Hint: Integration is the area under the curve.)

a.

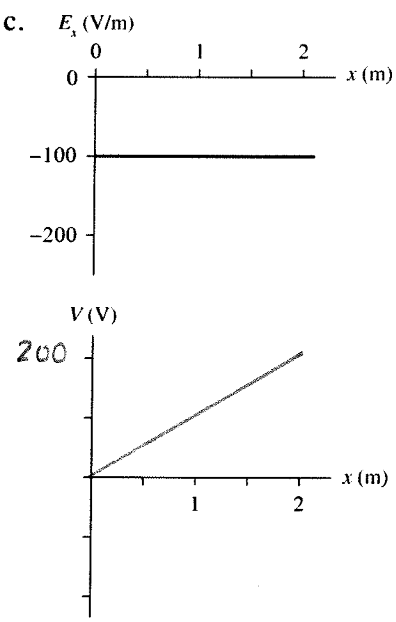


b.

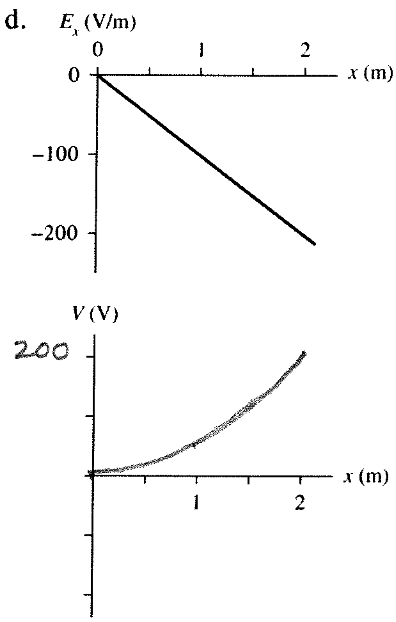


$$\Delta V = -\int_{x_i}^{x_f} E_x \, dx$$

c.



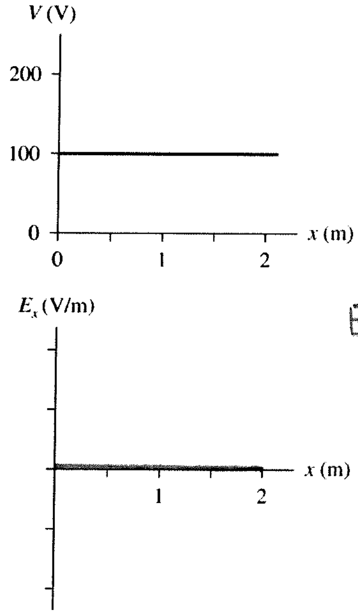
d.



26.2 Finding the Electric Field from the Potential

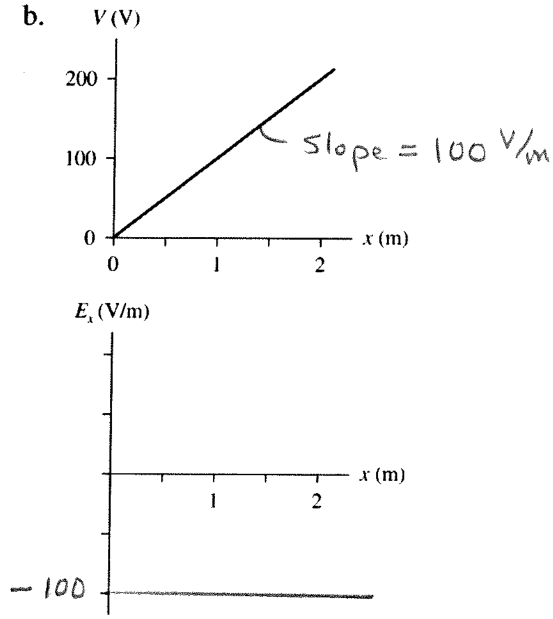
2. The top graph shows the electric potential as a function of x. On the axes below the graph, draw the graph of E_x versus x in this same region of space. Add an appropriate scale on the vertical axis.

a.



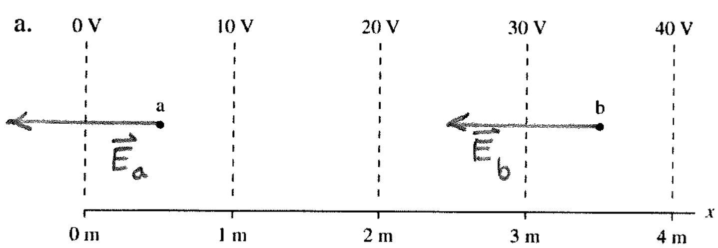
$$E_x = -\frac{\partial V}{\partial x}$$

b.



3. For each contour map:
   i. Estimate the electric fields $\vec{E}_a$ and $\vec{E}_b$ at points a and b. Don't forget that $\vec{E}$ is a vector. Show how you made your estimate.
   ii. On the contour map, draw the electric field vectors at points a and b.

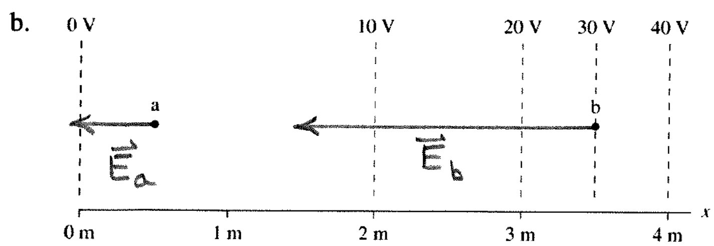
a.



$\vec{E}_a = -\frac{\Delta V}{\Delta x} = \frac{-(10-0)\text{V}}{1\text{m}} = -10\frac{\text{V}}{\text{m}}$

$\vec{E}_b = -10\ \text{V/m}$

b.

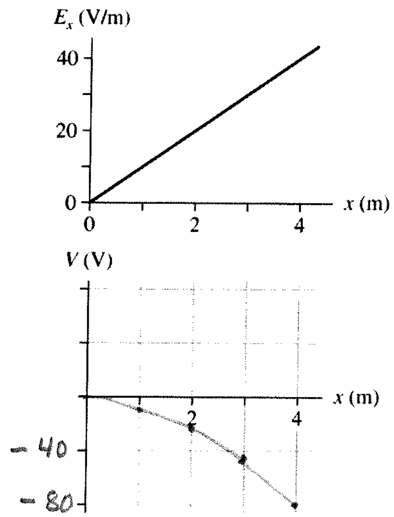


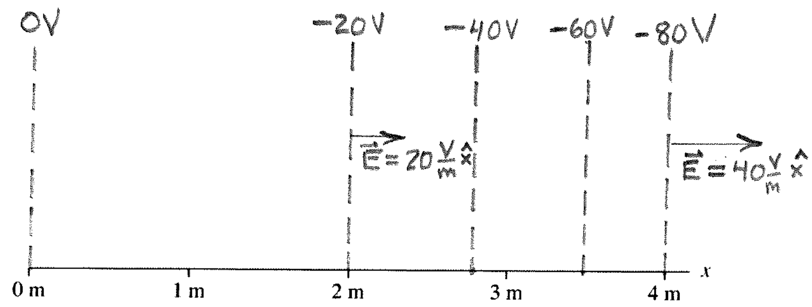
$\vec{E}_a = \frac{-(10-0)\text{V}}{2\text{m}-0\text{m}} = -5\frac{\text{V}}{\text{m}}$

$\vec{E}_b = -\left(\frac{40\text{V}-20\text{V}}{4\text{m}-3\text{m}}\right) = -20\frac{\text{V}}{\text{m}}$

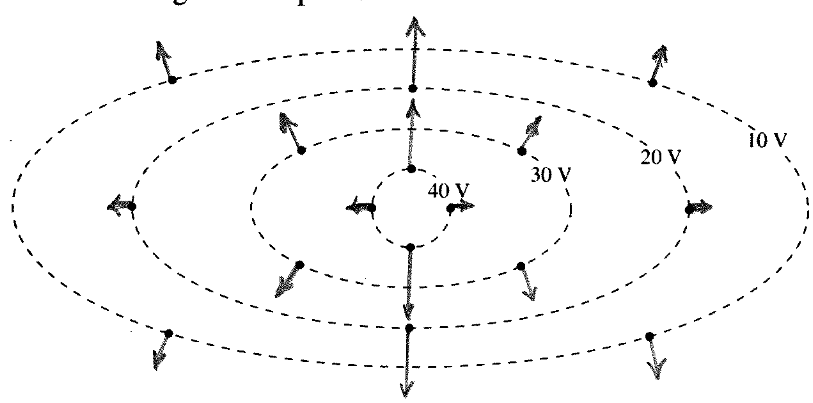
The minus sign means the electric field vector points "downhill" on the contour map.

4. The top graph shows E_x versus x for an electric field that is parallel to the x-axis.
   a. Draw the graph of V versus x in this region of space. Let $V = 0$ V at $x = 0$ m. Add an appropriate scale on the vertical axis. (Hint: Integration is the area under the curve.)
   b. Use dashed lines to draw a contour map above the x-axis on the right. Space your equipotential lines every 20 volts and label each equipotential line.
   c. Draw electric field vectors on top of the contour map.

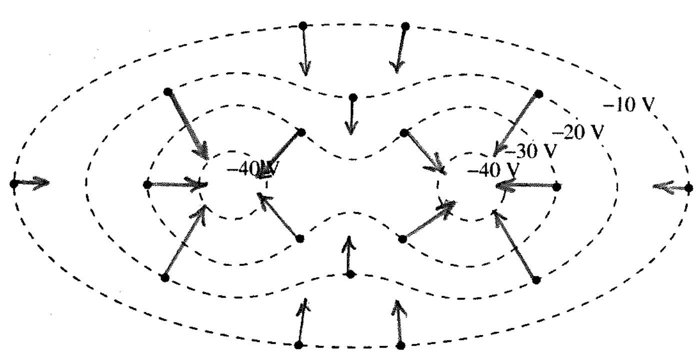




5. Draw the electric field vectors at the dots on this contour map. The length of each vector should be proportional to the field strength at that point.



6. Draw the electric field vectors at the dots on this contour map. The length of each vector should be proportional to the field strength at that point.



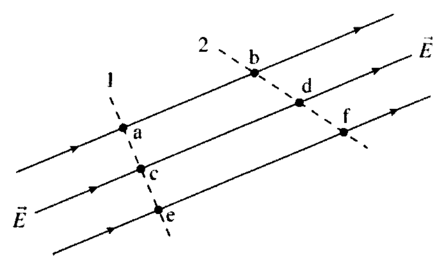
7. a. Suppose $\vec{E} = \vec{0}$ V/m throughout some region of space. Is $V = 0$ V in this region? Explain.

Not necessarily. The potential can have any value, but it cannot vary in this region of space since $E = -\frac{\partial V}{\partial s}$. $(\Delta V = 0V)$

b. Suppose $V = 0$ V throughout some region of space. Is $\vec{E} = \vec{0}$ V/m in this region? Explain.

Yes. If V has any constant value throughout a region of space, then the $\vec{E}$ field is $\vec{0}$ in that region.

8. The figure shows an electric field diagram. Dashed lines 1 and 2 are two surfaces in space, not physical objects.



a. Is the electric potential at point a higher than, lower than, or equal to the electric potential at point b? Explain.

$V_a > V_b$ since $\vec{E}$ points toward decreasing potential.

b. Rank in order, from largest to smallest, the magnitudes of potential differences ΔV_{ab}, ΔV_{cd}, and ΔV_{ef}.

Order: In magnitudes, $|\Delta V_{ef}| > |\Delta V_{cd}| > |\Delta V_{ab}|$

Explanation:

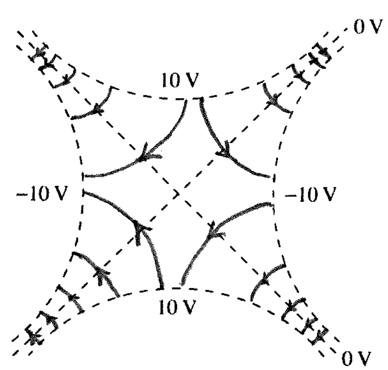
$|\Delta V| = |-E \Delta s|$ where E is uniform everywhere and where $\Delta s_{ef} > \Delta s_{cd} > \Delta s_{ab}$.

c. Is surface 1 an equipotential surface? What about surface 2? Explain why or why not.

Surface 1 is an equipotential surface, but not surface 2. An equipotential surface must be perpendicular to $\vec{E}$-field lines.

9. For each of the figures below, is this a physically possible potential map if there are no free charges in this region of space? If so, draw an electric field line diagram on top of the potential map. If not, why not?

a.



b.

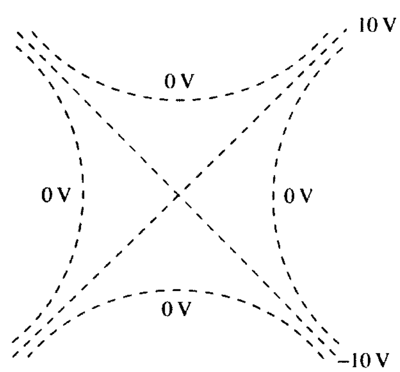
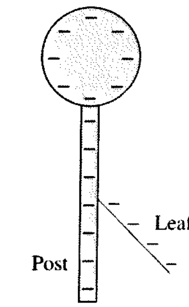


Figure b is not physically possible. Potential lines of different values cross at the center, but no space point can have two potential values.

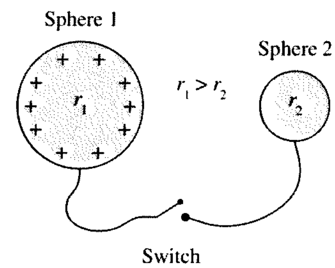
26.3 A Conductor in Electrostatic Equilibrium

10. The figure shows a negatively charged electroscope. The gold leaf stands away from the rigid metal post. Is the electric potential of the leaf higher than, lower than, or equal to the potential of the post? Explain.



Equal. When a conductor is in electrostatic equilibrium, the entire conductor is at the same potential.

11. Two metal spheres are connected by a metal wire that has a switch in the middle. Initially the switch is open. Sphere 1, with the larger radius, is given a positive charge. Sphere 2, with the smaller radius, is neutral. Then the switch is closed. Afterward, sphere 1 has charge Q_1, is at potential V_1, and the electric field strength at its surface is E_1. The values for sphere 2 are Q_2, V_2, and E_2.



a. Is V_1 larger than, smaller than, or equal to V_2? Explain.

$V_1 = V_2$. Both spheres and the wire become one conductor all at the same potential.

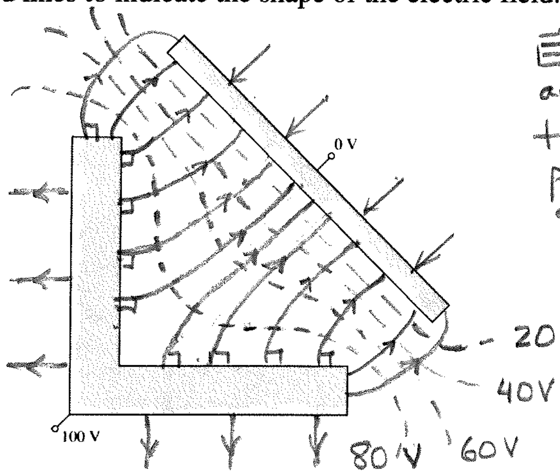
b. Is Q_1 larger than, smaller than, or equal to Q_2? Explain.

$Q_1 > Q_2$. Since $V_1 = V_2$ and $V \propto \frac{Q}{r}$, the sphere with the larger radius must hold more charge.

c. Is E_1 larger than, smaller than, or equal to E_2? Explain.

$E_1 < E_2$. Since $V_1 = V_2$ and $E = \frac{V}{r}$, the sphere with the larger radius must have a weaker electric field at the surface.

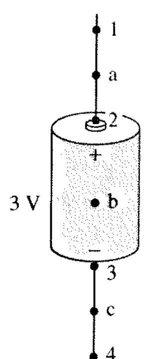
12. The figure shows two flat metal electrodes that are held at potentials of 100 V and 0 V.
    a. Used dashed lines to sketch a reasonable approximation of the 20 V, 40 V, 60 V, and 80 V equipotential lines.
    b. Draw enough electric field lines to indicate the shape of the electric field.



$\vec{E}$- field lines begin and end perpendicular to electrodes and are perpendicular to the equipotentials.

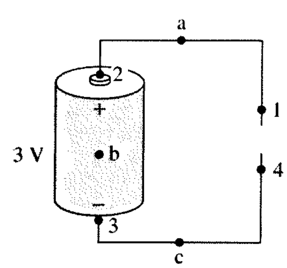
13. The figure shows two 3 V batteries with metal wires attached to each end. Points a and c are *inside* the wire. Point b is inside the battery. For each figure:
    - What are the potential differences ΔV_{12}, ΔV_{23}, ΔV_{34}, and ΔV_{14}? Define $\Delta V_{ab} = V_a - V_b$.
    - Does the electric field at a, b, and c point left, right, up, or down? Or is $\vec{E} = \vec{0}$?

a.



$\Delta V_{12} =$ 0V $\Delta V_{23} =$ 3 V
$\Delta V_{34} =$ 0V $\Delta V_{14} =$ 3 V
$\vec{E}_a$ $\vec{0}$ $\vec{E}_b$ down $\vec{E}_c$ $\vec{0}$

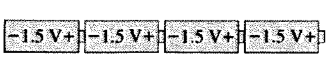
b.



$\Delta V_{12} =$ 0 V $\Delta V_{23} =$ 3 V
$\Delta V_{34} =$ 0 V $\Delta V_{14} =$ 3 V
$\vec{E}_a$ $\vec{0}$ $\vec{E}_b$ down $\vec{E}_c$ $\vec{0}$

26.4 Sources of Electric Potential

14. What is ΔV_{series} for each group of 1.5 V batteries?

a. 

$\Delta V_{series} =$ 6 V

b. 

$\Delta V_{series} =$ 3V

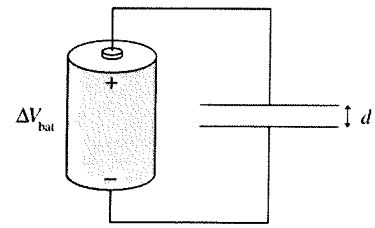
c. 

$\Delta V_{series} =$ 0V

26.5 Capacitance and Capacitors

26.6 The Energy Stored in a Capacitor

15. A parallel-plate capacitor with plate separation d is connected to a battery that has potential difference ΔV_{bat}. Without breaking any of the connections, insulating handles are used to increase the plate separation to $2d$.



a. Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why not?

No. The potential difference across the capacitor is determined by ΔV_{bat} and the battery is still the same.

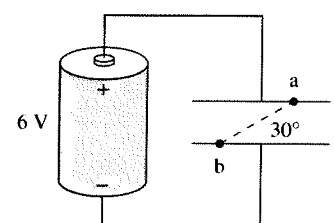
b. Does the capacitance change? If so, by what factor? If not, why not?

Yes. The capacitance depends on its geometry. For parallel-plates, $C = \frac{\epsilon_0 A}{d}$ where $d \rightarrow 2d$ so C decreases by a factor of 2.

c. Does the capacitor charge Q change? If so, by what factor? If not, why not?

Yes. Since $C = Q/\Delta V_C$ with no change in ΔV_C, Q must change by the same factor C changes. Q decreases by a factor of 2.

16. For the capacitor shown, the potential difference ΔV_{ab} between points a and b is

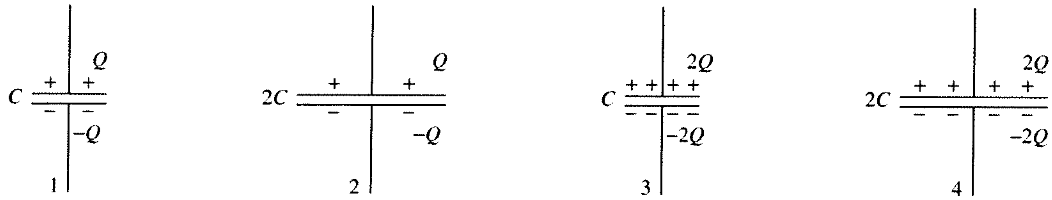


a. 6 V	b. 6 · sin 30° V	c. 6/sin 30° V
d. 6 · tan 30° V	e. 6 · cos 30° V	f. 6/cos 30° V

Explain your choice.

The entire top plate is at the same potential as the positive electrode of the battery (6V) and the entire bottom plate is at the same potential as the negative electrode (0V). Each capacitor plate is an equipotential surface.

17. Rank in order, from largest to smallest, the potential differences $(\Delta V_C)_1$ to $(\Delta V_C)_4$ of these four capacitors.



Order: $(\Delta V_C)_3 > (\Delta V_C)_1 = (\Delta V_C)_4 > (\Delta V_C)_2$

Explanation:

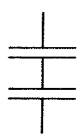
$$(\Delta V_C)_1 = \frac{Q}{C} = \frac{2Q}{2C} = (\Delta V_C)_4$$

$$(\Delta V_C)_3 = \frac{2Q}{C} = 2(\Delta V_1)$$

$$(\Delta V_C)_2 = \frac{Q}{2C} = \frac{1}{2}(\Delta V_1)$$

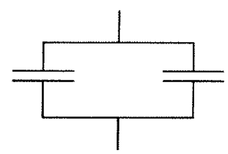
18. Each capacitor in the circuits below has capacitance C. What is the equivalent capacitance of the group of capacitors?

a.



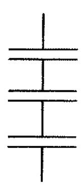
$C_{eq} =$ $C/2$

b.



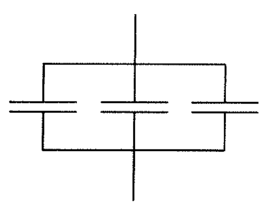
$C_{eq} =$ $2C$

c.



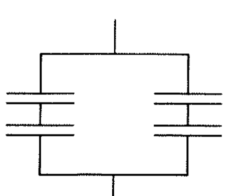
$C_{eq} =$ $C/3$

d.



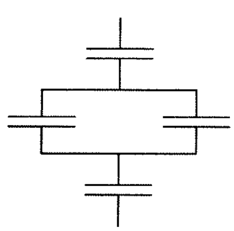
$C_{eq} =$ $3C$

e.



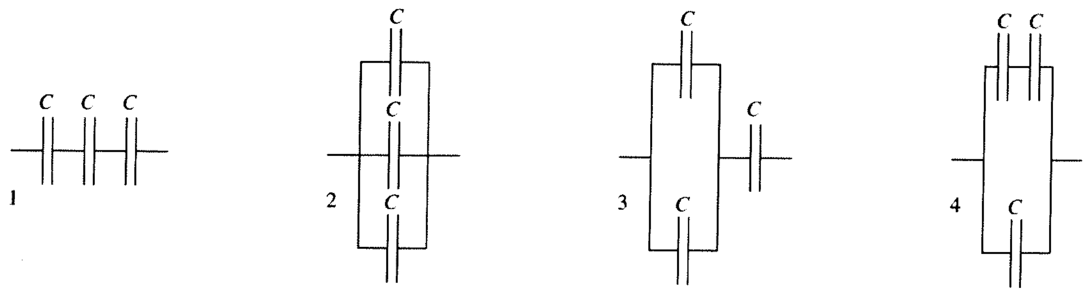
$C_{eq} =$ C

f.



$C_{eq} =$ $2C/5$

19. Rank in order, from largest to smallest, the equivalent capacitances $(C_{eq})_1$ to $(C_{eq})_4$ of these four groups of capacitors.



Order: $(C_{eq})_2 > (C_{eq})_4 > (C_{eq})_3 > (C_{eq})_1$

Explanation:

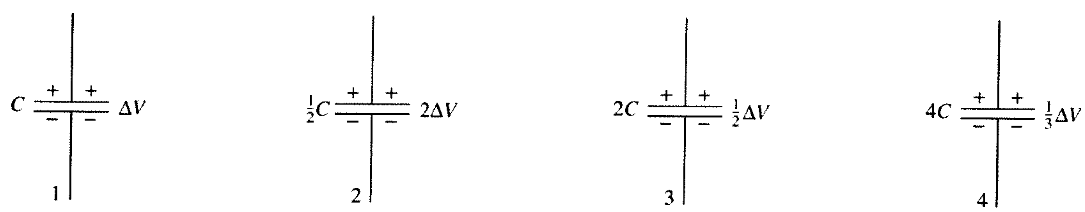
$$(C_{eq})_1 = \left(\frac{1}{C} + \frac{1}{C} + \frac{1}{C}\right)^{-1} = \frac{1}{3}C$$

$$(C_{eq})_2 = C + C + C = 3C$$

$$(C_{eq})_3 = \left(\frac{1}{2C} + \frac{1}{C}\right)^{-1} = \frac{2}{3}C$$

$$(C_{eq})_4 = \frac{C}{2} + C = \frac{3}{2}C$$

20. Rank in order, from largest to smallest, the energies $(U_C)_1$ to $(U_C)_4$ stored in each of these capacitors.



Order: $(U_c)_2 > (U_c)_1 > (U_c)_3 > (U_c)_4$

Explanation:

$$U_1 = \frac{1}{2}C(\Delta V)^2$$

$$U_2 = \frac{1}{2}\left(\frac{1}{2}C\right)(2\Delta V)^2 = 2U_1$$

$$U_3 = \frac{1}{2}(2C)\left(\frac{1}{2}\Delta V\right)^2 = \frac{1}{2}U_1$$

$$U_4 = \frac{1}{2}(4C)\left(\frac{1}{3}\Delta V\right)^2 = \frac{4}{9}U_1$$

26.7 Dielectrics

21. An air-insulated capacitor is charged until the electric field strength inside is 10,000 V/m, then disconnected from the battery. When a dielectric is inserted between the capacitor plates, filling the space, the electric field strength is reduced to 2000 V/m.

a. Does the amount of charge on the capacitor plates increase, decrease, or stay the same when the dielectric is inserted? If it increases or decreases, by what factor?

Stays the same. Charge is conserved, and with the capacitor disconnected from the battery the charge can go no where.

b. Does the potential difference between the capacitor plates increase, decrease, or stay the same when the dielectric is inserted? If it increases or decreases, by what factor?

The potential difference decreases by a factor of 5. The plates are still the same distance apart, but the field with the dielectric inserted between the plates is 5 times weaker ($\Delta V_c = E d$).

22. An air-insulated capacitor is charged until the plates have charge $\pm 10\ \mu C$, then left connected to the battery. When a dielectric is inserted between the capacitor plates, filling the space, the charge increases to $\pm 50\ \mu C$.

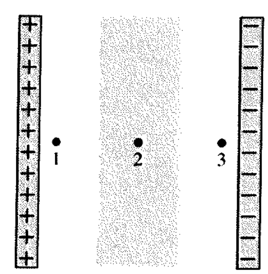
a. Does the potential difference across the capacitor plates increase, decrease, or stay the same when the capacitor is inserted? If it increases or decreases, by what factor?

The potential difference across the plates (ΔV_c) stays the same because it is fixed by the connection to the battery.

b. Does the electric field between the capacitor plates increase, decrease, or stay the same when the capacitor is inserted? If it increases or decreases, by what factor?

Stays the same since there is no change in potential difference across the plates or in plate separation, $E = \frac{\Delta V_c}{d}$. The additional field from the added charge to the plates is balanced out by the opposing field from the induced charge in the dielectric.

23. The gap between two capacitor plates is *partially* filled with a dielectric. Rank in order, from largest to smallest, the electric field strengths E_1, E_2, and E_3 at points 1, 2, and 3.



Order: $E_1 = E_2 > E_3$

Explanation:

Induced surface charges in the dielectric oppose the external field from the plates, so the field is weakened but only in the region of the dielectric.