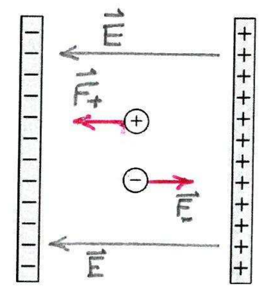
25 The Electric Potential

25.1 Electric Potential Energy

25.2 The Potential Energy of Point Charges

1. A positive point charge and a negative point charge are inside a parallel-plate capacitor. The point charges interact only with the capacitor, not with each other. Let the negative capacitor plate be the zero of potential energy for both charges.



   a. Use a **black** pen or pencil to draw the electric field vectors inside the capacitor.

   b. Use a **red** pen or pencil to draw the forces acting on the two charges.

   c. Is the potential energy of the *positive* point charge positive, negative, or zero? Explain.

   Positive. The potential energy $U = qEs$ where the charge q is positive, the electric field strength E is positive and the position s is positive relative to negative plate (where $s=0$).

   d. In which direction (right, left, up, or down) does the potential energy of the positive charge decrease? Explain.

   Left. As indicated above, the potential energy U is positive between the plates. It becomes less positive and approaches zero as the charge's position s from the negative plate approaches zero.

   e. In which direction will the positive charge move if released from rest? Use the concept of energy to explain your answer.

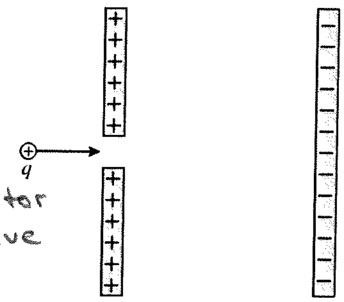
   Left. To maintain constant total energy, when released from rest the charge loses potential energy as it gains kinetic energy in its "fall" toward the negative plate.

   f. Does your answer to part e agree with the force vector you drew in part b? Yes

   g. Repeat steps c to f for the *negative* point charge.

   c. Negative. $U = qEs$ with E and s positive, but q negative.
   d. Right. The potential energy becomes more negative as the charge moves to the right.
   e. Right. The charge moves in the direction of decreasing potential energy as it gains kinectic energy.
   f. Yes.

2. A positive charge q is fired through a small hole in the positive plate of a capacitor. Does q speed up or slow down inside the capacitor? Answer this question twice:



a. First using the concept of force.

It speeds up. A positive charge inside the capacitor experiences an attractive force toward the negative plate and a repulsive force from the positive plate. Both forces are to the right.

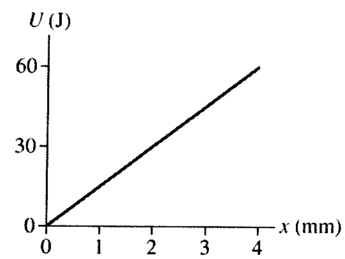
b. Second using the concept of energy.

It speeds up. After entering the capacitor the positive charge maintains constant total energy. So as the charge heads toward the negative plate, it loses potential energy but gains kinetic energy.

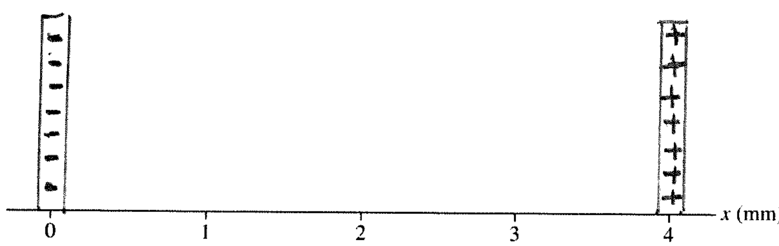
3. Charge $q_1 = 3$ nC is distance r from a positive point charge Q. Charge $q_2 = 1$ nC is distance $2r$ from Q. What is the ratio U_1/U_2 of their potential energies due to their interactions with Q?

$$\frac{U_1}{U_2} = \frac{\frac{1}{4\pi\epsilon_0}\frac{q_1 Q}{r}}{\frac{1}{4\pi\epsilon_0}\frac{q_2 Q}{2r}} = \frac{q_1(2r)}{q_2(r)} = \frac{3\,nC\ (2r)}{1\,nC\ (r)} = 6$$

4. The figure shows the potential energy of a positively charged particle in a region of space.



a. What possible arrangement of source charges is responsible for this potential energy? Draw the source charges above the axis below.



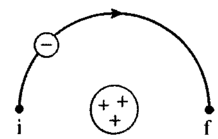
b. With what kinetic energy should the charged particle be launched from $x = 0$ mm to have a turning point at $x = 3$ mm? Explain.

K = 45 J initially (at x = 0 mm). The total energy of the charge is constant. The graph shows an increase of 45 J of potential energy U as the charge moves from x = 0 mm to x = 3 mm. This gain in U must from a loss in K. K is zero at x = 3 mm (turning point).

c. How much kinetic energy does this charged particle of part b have as it passes $x = 2$ mm?

15 J. The graph shows U increases by 30 J so the loss in K must be 30 J.

5. An electron ($q = -e$) completes half of a circular orbit of radius r around a nucleus with $Q = +3e$.



a. How much work is done on the electron as it moves from i to f? Give either a numerical value or an expression from which you could calculate the value if you knew the radius. Justify your answer.

Zero work is done. The force is always perpendicular to the motion.

b. By how much does the electric potential energy change as the electron moves from i to f?

No change. The change in potential energy is negative of the work done by the electric force, but no work is done during this motion.

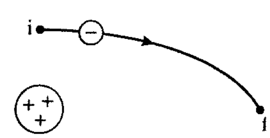
c. Is the electron's speed at f greater than, less than, or equal to its speed at i?

Equal. Since the total work done is zero, the kinetic energy and speed do not change.

d. Are your answers to parts a and c consistent with each other?

Yes.

6. An electron moves along the trajectory from i to f.



a. Does the electric potential energy increase, decrease, or stay the same? Explain.

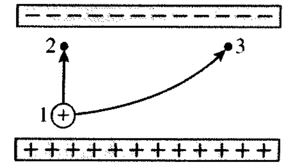
$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$ where r = separation distance

The potential energy is proportional to $\frac{1}{r}$, but since the charges have opposite sign the potential energy increases (becomes less negative) as r increases from i to f.

b. Is the electron's speed at f greater than, less than, or equal to its speed at i? Explain.

Less than. As the potential energy increases the kinetic energy and speed decrease since the total energy must remain constant.

7. Inside a parallel-plate capacitor, two protons are launched with the same speed from point 1. One proton moves along the path from 1 to 2, the other from 1 to 3. Points 2 and 3 are the same distance from the negative plate.



a. Is $\Delta U_{1\to 2}$, the change in potential energy along the path $1 \to 2$, larger than, smaller than, or equal to $\Delta U_{1\to 3}$? Explain.

$\Delta U_{1\to 2} = \Delta U_{1\to 3}$

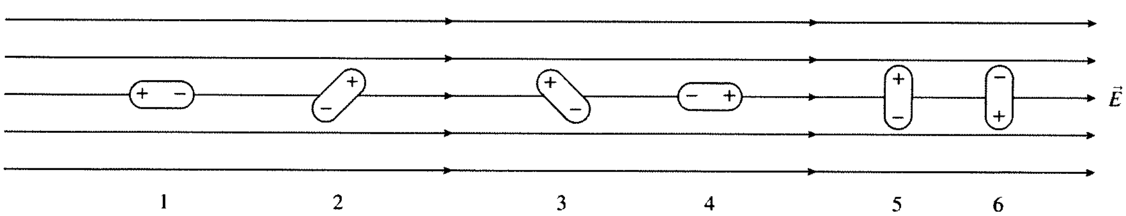
$\Delta U = q E \Delta s$ where Δs is the change in vertical position from point 1 to point 2 or point 3. $\Delta s_{1\to 2} = \Delta s_{2\to 3}$.
Only vertical displacement matters here since there is no horizontal component of $\vec{E}$ or the electric force.

b. Is the proton's speed v_2 at point 2 larger than, smaller than, or equal to v_3? Explain.

$V_2 = V_3$. Both paths have the same ΔK since both have the same ΔU and total energy must be constant.

25.3 The Potential Energy of a Dipole

8. Rank in order, from most positive to most negative, the potential energies U_1 to U_6 of these six electric dipoles in a uniform electric field.

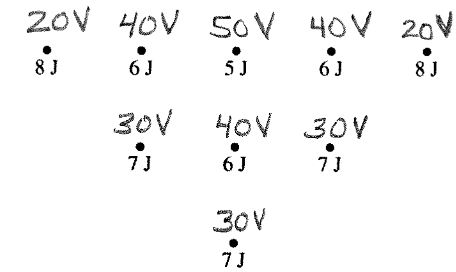


Order: $U_1 > U_3 > U_6 = U_5 > U_2 > U_4$

Explanation:
The potential energy U of a dipole depends on the work required to orient the dipole in the field. U is most negative when the dipole aligns with the field (as in 4) but is most positive when dipole is opposite the field (as in 1). Dipole 3 somewhat opposes the field (so $U>0$), dipoles 6 and 5 are midway (and have $U=0$), and dipole 2 is somewhat aligned with the field (so $U<0$).

25.4 The Electric Potential

9. Charged particles with $q = +0.1$ C are fired with 10 J of kinetic energy toward a region of space in which there is an electric potential. The figure shows the kinetic energy of the charged particles as they arrive at nine different points in the region. Determine the electric potential at each of these points. Write the value of the potential *above* each of the dots. Assume that the particles start from a point where the electric potential is zero.



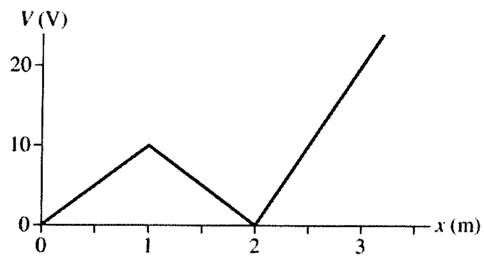
Example:

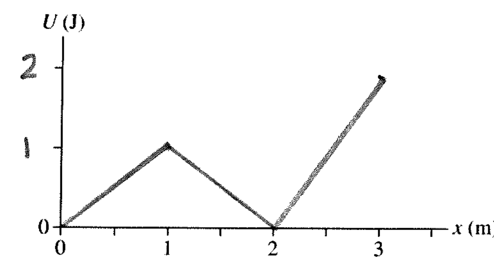
Total Energy = $K + U = 10J$

If $K = 7J$ then $U = 3J = qV$

so $V = \frac{U}{q} = \frac{3J}{0.1C} = 30$ Volts

10. a. The graph on the left shows the electric potential along the x-axis. Use the axes on the right to draw a graph of the potential energy of a 0.1 C charged particle in this region of space. Provide a numerical scale on the energy axis.





b. If the charged particle is shot toward the right from $x = 1$ m with 1.0 J of kinetic energy, where is its turning point? Explain.

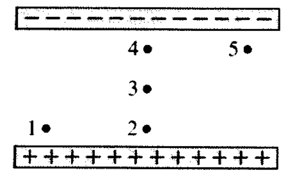
At $x = 1$m, the total energy $E = K_1 + U_1 = 1J + 1J = 2J$.
At turning point $v = 0$ so $K_2 = 0$ and
$E = K_2 + U_2 = 2J$ so $U_2 = 2J$ which occurs at $x = 3$m.
Turning point is at $x = 3$m.

c. Will the charged particle of part b ever reach $x = 0$ m? If so, how much kinetic energy will it have at that point? If not, why not?

Yes. Since $E = 2J$ (constant) and $U = 0J$ at $x = 0$m, we know $K = 2J$.

25.5 The Electric Potential Inside a Parallel-Plate Capacitor

11. Rank in order, from largest to smallest, the electric potentials V_1 to V_5 at points 1 to 5.

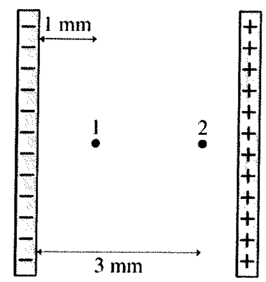


Order: $V_1 = V_2 > V_3 > V_4 = V_5$

Explanation:

$V = Es$ where s is measured from the negative plate.

12. The figure shows two points inside a capacitor. Let $V = 0$ V at the negative plate.



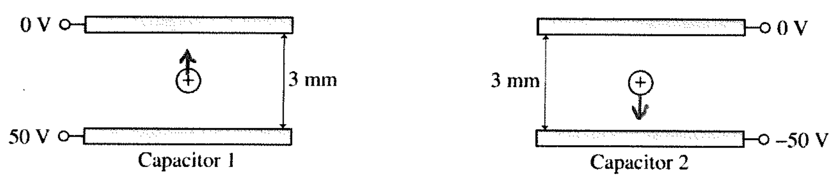
a. What is the ratio V_2/V_1 of the electric potentials at these two points? Explain.

$$\frac{V_2}{V_1} = \frac{Es_2}{Es_1} = \frac{s_2}{s_1} = \frac{3\text{mm}}{1\text{mm}} = 3$$

b. What is the ratio E_2/E_1 of the electric field strengths at these two points? Explain.

$\frac{E_2}{E_1} = 1$ The E-field is constant between the plates.

13. The figure shows two capacitors, each with a 3 mm separation. A proton is released from rest in the center of each capacitor.



a. Draw an arrow on each proton to show the direction it moves.

b. Which proton reaches a capacitor plate first? Or are they simultaneous? Explain.

They reach the plate simultaneously. The electric field strength is the same, So the size of acceleration is the same.

14. A capacitor with plates separated by distance d is charged to a potential difference ΔV_C. All wires and batteries are disconnected, then the two plates are pulled apart (with insulated handles) to a new separation of distance $2d$.

a. Does the capacitor charge Q change as the separation increases? If so, by what factor? If not, why not?

No. Q is constant. With the use of insulated handles no charge will flow off the plates.

b. Does the electric field strength E change as the separation increases? If so, by what factor? If not, why not?

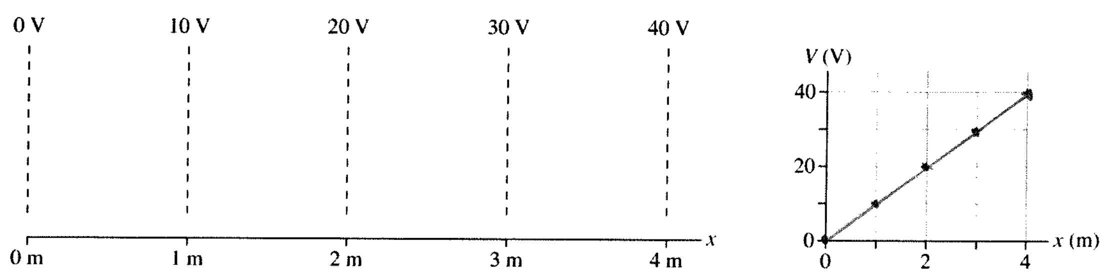
No. E is constant. Let A be the surface area of one of the plates, then $E = \frac{\eta}{\epsilon_0} = \frac{Q/A}{\epsilon_0}$ = constant.

c. Does the potential difference ΔV_C change as the separation increases? If so, by what factor? If not, why not?

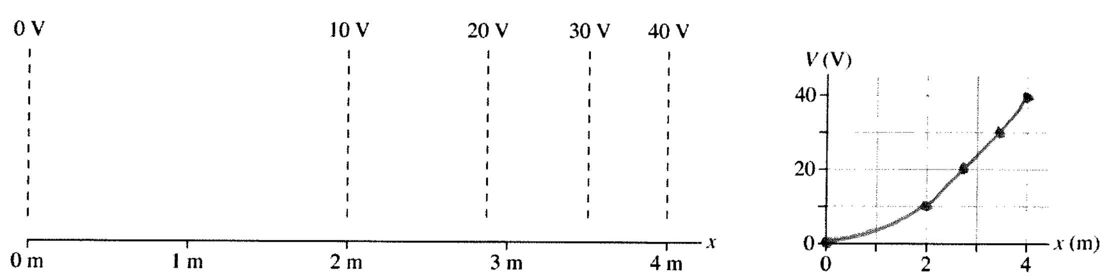
Yes. ΔV_C increases as the separation increases.
$\Delta V_C = Ed$ so ΔV_C increases by a factor of 2 when d doubles.

15. Each figure shows a contour map on the left and a set of graph axes on the right. Draw a graph of V versus x. Your graph should be a straight line or a smooth curve.

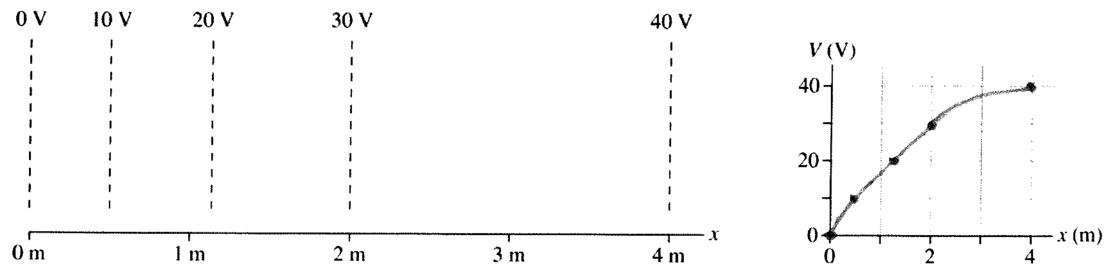
a.



b.

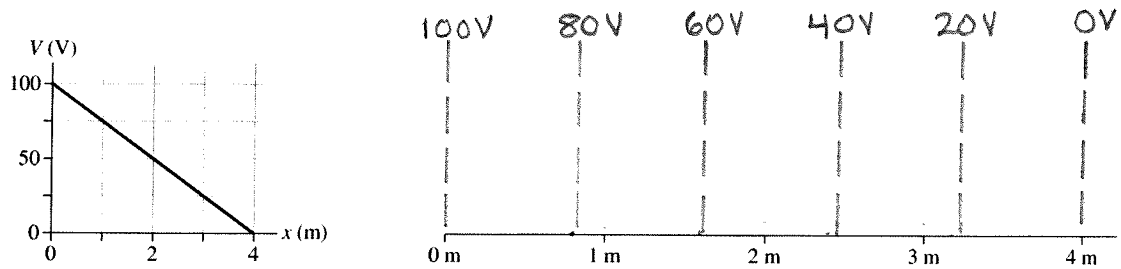


c.

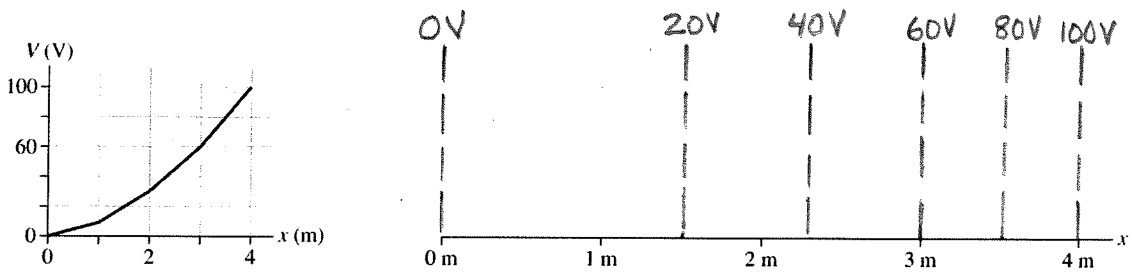


16. Each figure shows a V-versus-x graph on the left and an x-axis on the right. Assume that the potential varies with x but not with y. Draw a contour map of the electric potential. Your figures should look similar to the contour maps in Question 15. Space your equipotential lines every 20 volts and label them.

a.

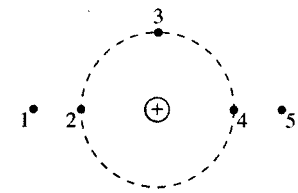


b.



25.6 The Electric Potential of a Point Charge

17. Rank in order, from largest to smallest, the electric potentials V_1 to V_5 at points 1 to 5.



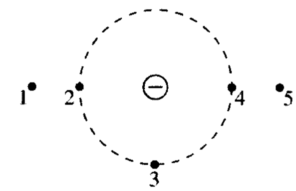
Order: $V_2 = V_3 = V_4 > V_1 = V_5$

Explanation:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

As r increases, V decreases

18. Rank in order, from least negative to most negative, the electric potentials V_1 to V_5 at points 1 to 5.



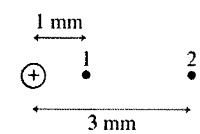
Order: $V_1 = V_5 > V_2 = V_3 = V_4$

Explanation:

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$ where q is negative.

As r increases, V increases (becomes less negative).

19. The figure shows two points near a positive point charge.



a. What is the ratio V_1/V_2 of the electric potentials at these two points? Explain.

V is inversely proportional to r.

So since $r_1 = r_2/3$, then $V_1 = 3V_2$.

$$\frac{V_1}{V_2} = 3.$$

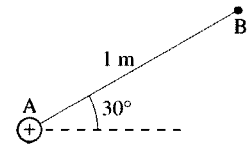
b. What is the ratio E_1/E_2 of the electric field strengths at these two points? Explain.

E is inversely proportional to r^2.

So since $r_1 = r_2/3$, then $E_1 = (3)^2 E_2$.

$$\frac{E_1}{E_2} = 9.$$

20. A 1 nC positive point charge is located at point A. The electric potential at point B is



a. 9 V b. $9 \cdot \sin 30°$ V c. $9 \cdot \cos 30°$ V d. $9 \cdot \tan 30°$ V

Explain the reason for your choice.

The radial distance from the given point charge is 1m. The given angle is not relevant.

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r} = \frac{(9.0\times10^9\ \frac{Nm^2}{C^2})(1.0\times10^{-9}C)}{1m} = 9V$$

21. An inflatable metal balloon of radius R is charged to a potential of 1000 V. After all wires and batteries are disconnected, the balloon is inflated to a new radius $2R$.

a. Does the potential of the balloon change as it is inflated? If so, by what factor? If not, why not?

Yes. The potential decreases by a factor of 2. Charge Q is constant.

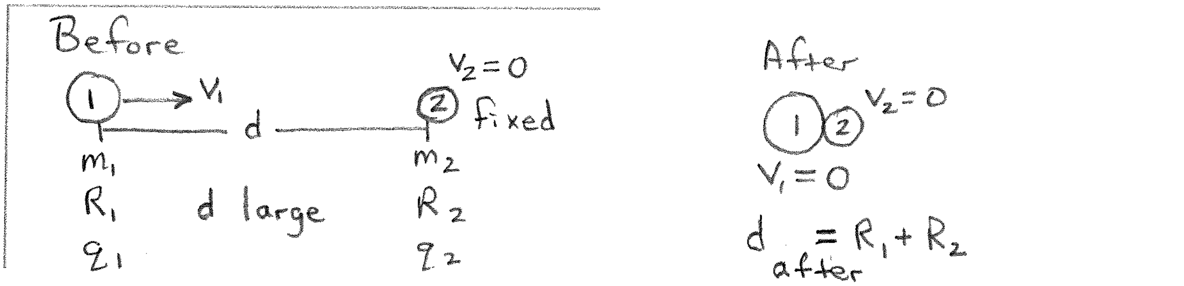
$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} \text{ and } V_2 = \frac{1}{4\pi\epsilon_0}\frac{Q}{2R}.$$

b. Does the potential at a point at distance $r = 4R$ change as the balloon is inflated? If so, by what factor? If not, why not?

No. Outside the sphere the potential is the same as that of a point charge Q located at the center of the sphere. The distance $r = 4R$ is always outside the balloon as it is inflated, and Q is unchanged.

22. A small charged sphere of radius R_1, mass m_1, and positive charge q_1 is shot head on with speed v_1 from a long distance away toward a second small sphere having radius R_2, mass m_2, and positive charge q_2. The second sphere is held in a fixed location and cannot move. The spheres repel each other, so sphere 1 will slow as it approaches sphere 2. If v_1 is small, sphere 1 will reach a closest point, reverse direction, and be pushed away by sphere 2. If v_1 is large, sphere 1 will crash into sphere 2. For what speed v_1 does sphere 1 just barely touch sphere 2 as it reverses direction?
PSS 25.1

a. Begin by drawing a before-and-after pictorial representation. Initially, the spheres are far apart and sphere 1 is heading toward sphere 2 with speed v_1. The problem ends with the spheres touching. What is speed of sphere 1 at this instant? How far apart are the centers of the spheres at this instant? Label the before and after pictures with complete information—all in symbolic form.



b. Energy is conserved, so we can use Problem-Solving Strategy 25.1. But first we have to identify the "moving charge" q and the "source charge" that creates the potential.

Which is the moving charge? q_1 Which is the source charge? q_2

c. We're told the charges start "a long distance away" from each other. Based on this statement, what value can you assign to V_i, the potential of the source charge at the initial position of the moving charge? Explain.

$V_i \approx 0$ since $V = \frac{1}{4\pi\epsilon_0}\frac{q_2}{d}$ and d is very large

d. Now write an expression in terms of the symbols defined above (and any constants that are needed) for the initial energy $K_i + qV_i$.

$K_i + qV_i = \frac{1}{2} m_1 v_1^2 + 0$

e. Referring to information on your visual overview, write an expression for the final energy.

$K_f + qV_f = 0 + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{(R_1 + R_2)}$

f. Energy is conserved, so finish the problem by solving for v_1.

Since energy is conserved,

$$\frac{1}{2} m_1 v_1^2 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{(R_1 + R_2)}.$$

Speed $$v_1 = \left[\frac{q_1 q_2}{2\pi\epsilon_0 m_1 (R_1 + R_2)}\right]^{1/2}.$$

25.7 The Electric Potential of Many Charges

23. Each figure below shows three points in the vicinity of two point charges. The charges have equal magnitudes. Rank in order, from largest to smallest, the potentials V_1, V_2, and V_3.

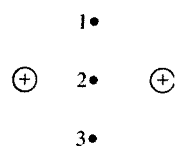
a.

$V_2 > V_1 = V_3$

b.

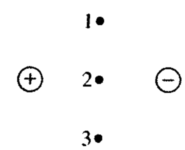
$V_1 > V_2 > V_3$

c.



$V_2 > V_1 = V_3$

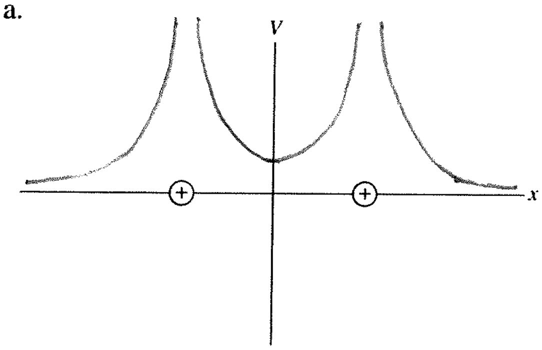
d.



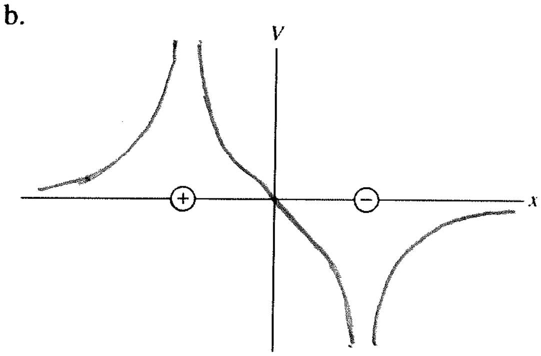
$V_1 = V_2 = V_3 = 0$

24. On the axes below, draw a graph of V versus x for the two point charges shown.

a.



b.



25. For each pair of charges below, are there any points (other than at infinity) at which the electric potential is zero? If so, show them on the figure with a dot and a $V = 0$ label. If not, why not?

a.

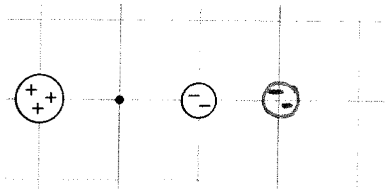
No. The potential is always positive in the given range. $V \to 0$ as $r \to \infty$.

b.

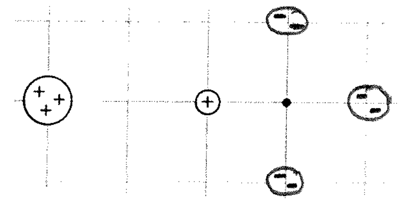
$V = 0$ $V = 0$

26. For each pair of charges below, at which grid point or points could a double-negative point charge ($q = -2$) be placed so that the potential at the dot is 0 V? There may be more than one possible point. Draw the charge on the figure at all points that work.

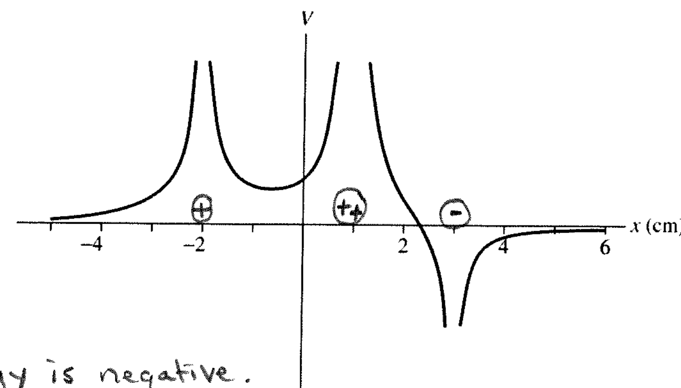
a.



b.



27. The graph shows the electric potential along the x-axis due to point charges on the x-axis.



a. Draw the charges *on the axis of the figure*. Note that the charges may have different magnitudes.

b. An electron is placed at $x = 2$ cm. Is its potential energy positive, negative, or zero? Explain.

The electron's potential energy is negative. $U = qV$ where q is negative and V is positive at $x = 2$cm.

c. If the electron is released from rest at $x = 2$ cm, will it move right, move left, or remain at $x = 2$ cm? Base your explanation on energy concepts.

The electron will move left. A charge will move in the direction of decreasing potential energy. The electron's potential energy becomes more negative as it moves left.

28. A ring has radius R and charge Q. The ring is shrunk to a new radius $\frac{1}{2}R$ with no change in its charge. By what factor does the on-axis potential at $z = R$ increase?

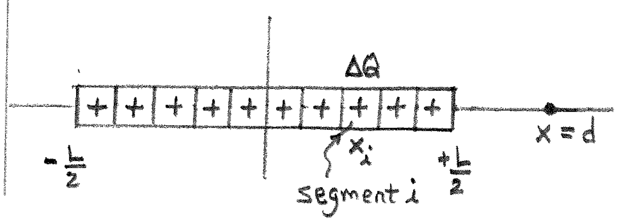
$$V = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{R^2+z^2}} \quad \text{so} \quad V_n \propto \frac{1}{\sqrt{R_n^2+z^2}} \quad \text{where} \quad \begin{array}{l} n = 1 \text{ or } 2 \\ R_1 = R \\ R_2 = R/2 \\ z = R \end{array}$$

$$\frac{V_2}{V_1} = \frac{\sqrt{R_1^2+z^2}}{\sqrt{R_2^2+z^2}} = \frac{\sqrt{R^2+R^2}}{\sqrt{R^2/4+R^2}} = \frac{\sqrt{2}}{\sqrt{5/4}} \approx 1.26$$

V increases by a factor of 1.26.

29. A thin rod of length L is uniformly charged with total charge Q. What is the electric potential on the axis of the rod at distance d from its center?
PSS 25.2

a. Begin with a visual representation. Draw a horizontal rod, then divide it into 10 or 12 boxes with a + in each box. Add an x-axis with the rod centered at the origin. Label the ends of the rod $x = -L/2$ and $x = L/2$. Put a dot on the x-axis at some point to the right of the rod; label it $x = d$.



b. Pick one of your + boxes to the right of the origin; label it "segment i," label its position as x_i, and write ΔQ beside it to show the charge in segment i.

c. Using what you know about the electric potential of a point charge, write an expression for the electric potential of segment i at the position of the dot. Your expression should be in terms of ΔQ, x_i, d, and various constants.

$$V_i = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q}{d - x_i}$$

d. Write an expression for the potential of the rod as a sum over all i of your answer to part c.

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{\Delta Q}{d - x_i}$$

e. The rod has charge Q in length L. What is the linear charge density? $\lambda =$ Q/L

f. Segment i has width Δx. Based on λ and Δx, the segment has charge $\Delta Q =$ $\lambda \Delta x$

g. Rewrite your answer to part d with this substitution for ΔQ.

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{\lambda \Delta x}{d - x_i}$$

h. Now you're ready to convert the sum to an integral. What are the integration limits?

Lower limit $-L/2$ Upper limit $+L/2$

i. Write your expression for the potential as a definite integral. That means (a) Change Δx to dx, (b) drop the subscript from x_i because x is now a continuous variable, (c) show the integration limits, and (d) take all multiplicative constants outside the integration.

$$V = \frac{\lambda}{4\pi\epsilon_0}\int_{-L/2}^{L/2} \frac{dx}{d - x}$$

We're going to stop here. You've done the physics by figuring out what to integrate. Now it's "just" a calculus problem of carrying out the integration to get a final answer.