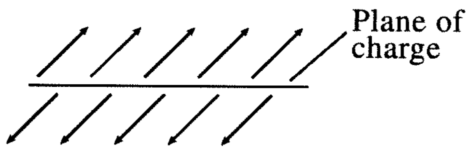
24 Gauss's Law

24.1 Symmetry

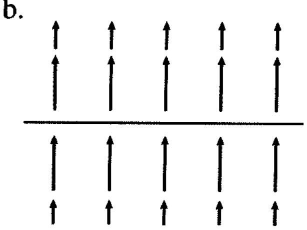
1. An infinite plane of charge is seen edge on. The sign of the charge is not given. Do the electric fields shown below have the same symmetry as the charge? If not, why not?

a.



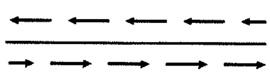
No. Reflection in the given plane shows the same charge but a different field.

b.



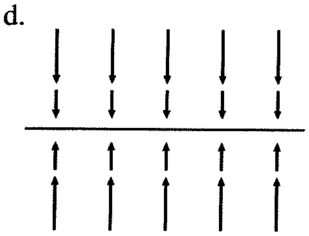
No. Same reasoning as (a).

c.



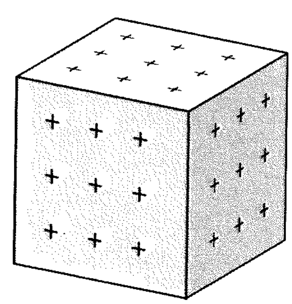
No. Same reasoning as (a).

d.



Yes. This field has the same symmetry as the charge.

2. Suppose you had a uniformly charged cube. Can you use symmetry alone to deduce the shape of the cube's electric field? If so, sketch and describe the field shape. If not, why not?



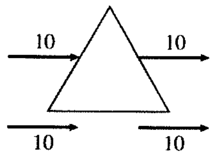
No. The cube lacks sufficient symmetry to deduce the shape of the field. In particular, the cube lacks both translational and rotational symmetry. It is symmetric under 90° rotations, but unlike a sphere, not symmetric for arbitrary rotations.

24.2 The Concept of Flux

3. The figures shown below are cross sections of three-dimensional closed surfaces. They have a flat top and bottom surface above and below the plane of the page. However, the electric field is everywhere parallel to the page, so there is no flux through the top or bottom surface. The electric field is uniform over each face of the surface. The field strength, in N/C, is shown.

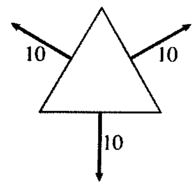
For each, does the surface enclose a net positive charge, a net negative charge, or no net charge?

a.



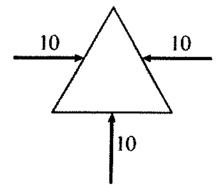
Q_{in} = O

b.



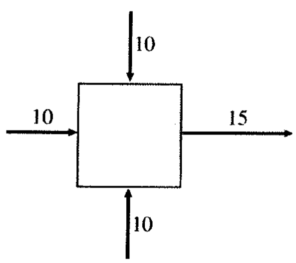
Q_{in} = +

c.



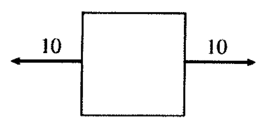
Q_{in} = –

d.



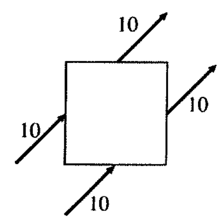
Q_{in} = –

e.



Q_{in} = +

f.

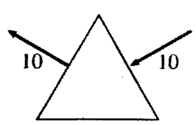


Q_{in} = O

4. The figures shown below are cross sections of three-dimensional closed surfaces. They have a flat top and bottom surface above and below the plane of the page, but there is no flux through the top or bottom surface. The electric field is perpendicular to and uniform over each face of the surface. The field strength, in N/C, is shown.

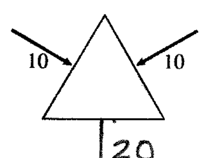
Each surface contains no net charge. Draw the missing electric field vector (or write $\vec{E} = \vec{0}$) in the proper direction. Write the field strength beside it.

a.

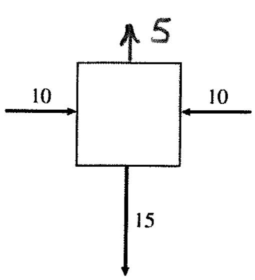


$\vec{E} = \vec{0}$

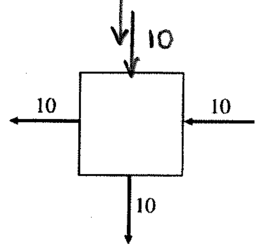
b.



c.

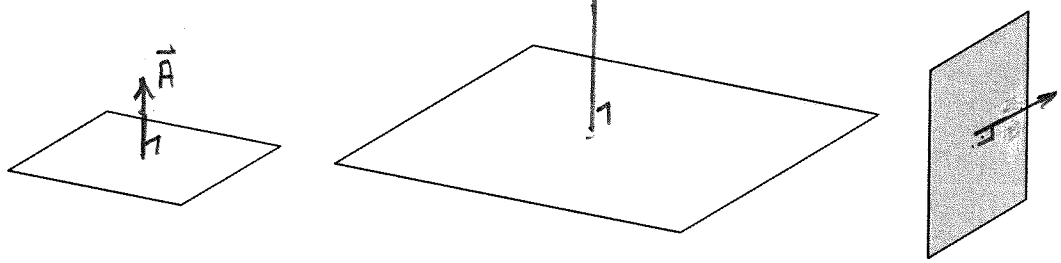


d.



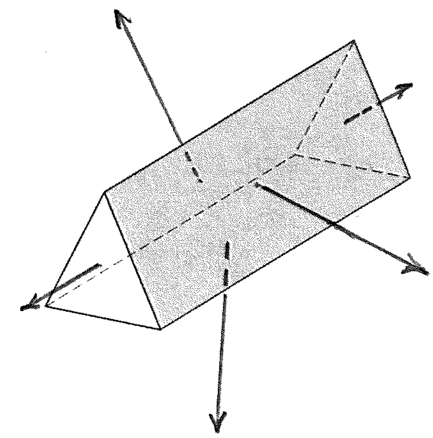
24.3 Calculating Electric Flux

5. Draw the area vector $\vec{A}$ for each of these surfaces.

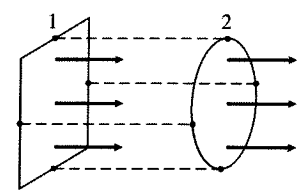


6. How many area vectors are needed to characterize this closed surface? 5

   Draw them.



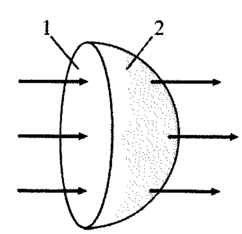
7. The diameter of the circle equals the edge length of the square. They are in a uniform electric field. Is the electric flux Φ_1 through the square larger than, smaller than, or equal to the electric flux Φ_2 through the circle? Explain.



Because $A_1 > A_2$ and $E_1 = E_2$

$\Phi_1 > \Phi_2$ where $\Phi = EA$

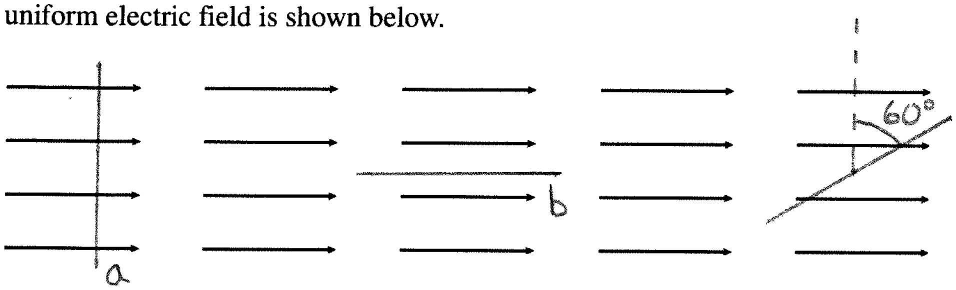
8. Is the electric flux Φ_1 through the circle larger than, smaller than, or equal to the electric flux Φ_2 through the hemisphere? Explain.



Any $\vec{E}$ line passing through surface 1 must also pass through surface 2.

$\Phi_1 = \Phi_2$

9. A uniform electric field is shown below.



Draw and label an *edge view* of three square surfaces, all the *same size*, for which

a. The flux is maximum.

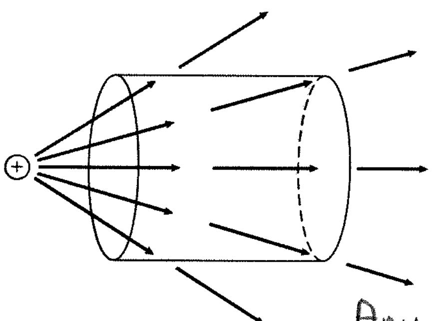
b. The flux is minimum.

c. The flux has half the value of the flux through the square part of a.

Give the tilt angle of any squares not perpendicular or parallel to the field lines.

10. Is the net electric flux through each of the closed surfaces below positive (+), negative (−), or zero (0)?

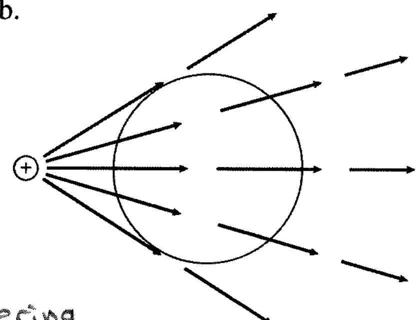
a.



$\Phi_e =$ 0

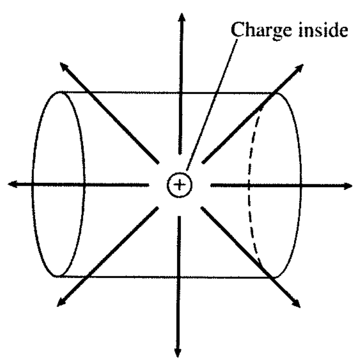
Any line entering also exits

b.



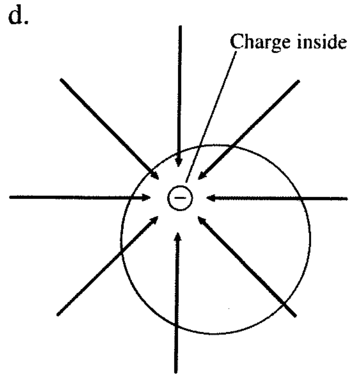
$\Phi_e =$ 0

c.



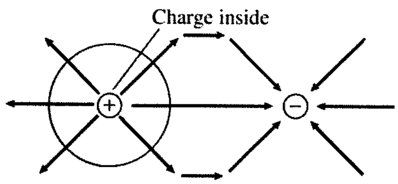
$\Phi_e =$ +

d.



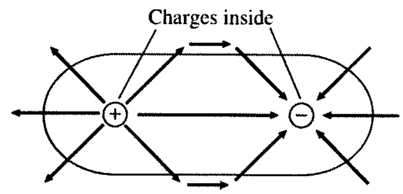
$\Phi_e =$ −

e.



$\Phi_e =$ +

f.



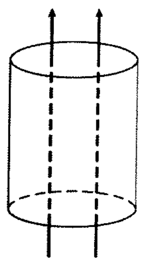
$\Phi_e =$ 0

24.4 Gauss's Law

24.5 Using Gauss's Law

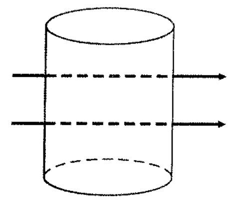
11. For each of the closed cylinders shown below, are the electric fluxes through the top, the wall, and the bottom positive (+), negative (−), or zero (0)? Is the net flux positive, negative, or zero?

a.



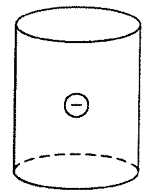
$\Phi_{top} =$ +
$\Phi_{wall} =$ 0
$\Phi_{bot} =$ −
$\Phi_{net} =$ 0

b.



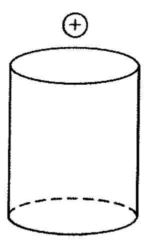
$\Phi_{top} =$ 0
$\Phi_{wall} =$ 0
$\Phi_{bot} =$ 0
$\Phi_{net} =$ 0

c.



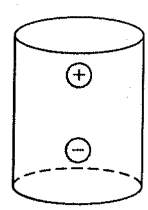
$\Phi_{top} =$ −
$\Phi_{wall} =$ −
$\Phi_{bot} =$ −
$\Phi_{net} =$ −

d.



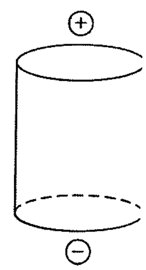
$\Phi_{top} =$ −
$\Phi_{wall} =$ +
$\Phi_{bot} =$ +
$\Phi_{net} =$ 0

e.



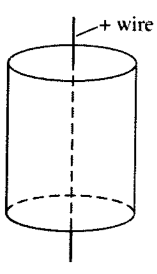
$\Phi_{top} =$ +
$\Phi_{wall} =$ 0
$\Phi_{bot} =$ −
$\Phi_{net} =$ 0

f.



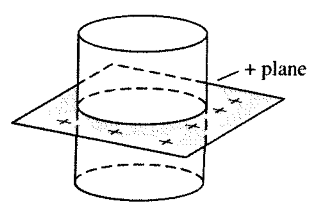
$\Phi_{top} =$ −
$\Phi_{wall} =$ 0
$\Phi_{bot} =$ +
$\Phi_{net} =$ 0

g.



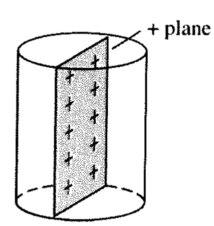
$\Phi_{top} =$ 0
$\Phi_{wall} =$ +
$\Phi_{bot} =$ 0
$\Phi_{net} =$ +

h.



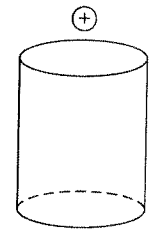
$\Phi_{top} =$ +
$\Phi_{wall} =$ 0
$\Phi_{bot} =$ +
$\Phi_{net} =$ +

i.



$\Phi_{top} =$ 0
$\Phi_{wall} =$ +
$\Phi_{bot} =$ 0
$\Phi_{net} =$ +

12. For this closed cylinder, $\Phi_{top} = -15\ \text{Nm}^2/\text{C}$ and $\Phi_{bot} = 5\ \text{Nm}^2/\text{C}$. What is Φ_{wall}?

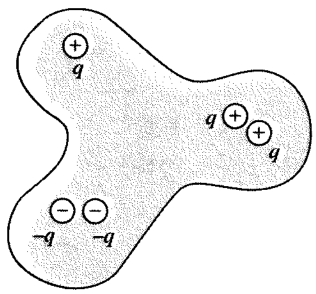


$\Phi_{wall} = 10\ \text{Nm}^2/\text{C}$

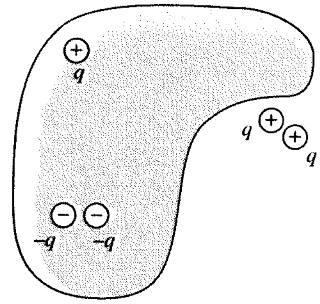
since $\Phi_{top} + \Phi_{bot} + \Phi_{wall} = 0$

13. What is the electric flux through each of these closed surfaces? (The charges in the gray areas are inside the closed surfaces.) Give your answers as multiples of q/ϵ_0.

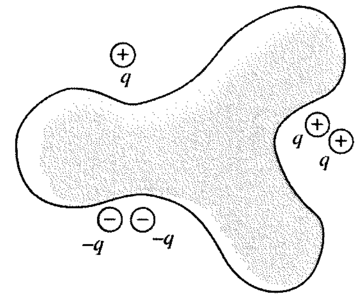
a.



b.

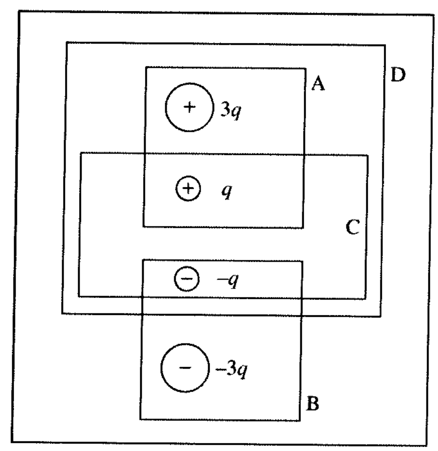


c.



$\Phi_e = q/\epsilon_0$ $\Phi_e = -q/\epsilon_0$ $\Phi_e = 0$

14. What is the electric flux through each of these surfaces? Give your answers as multiples of q/ϵ_0.



$\Phi_A = 4q/\epsilon_0$

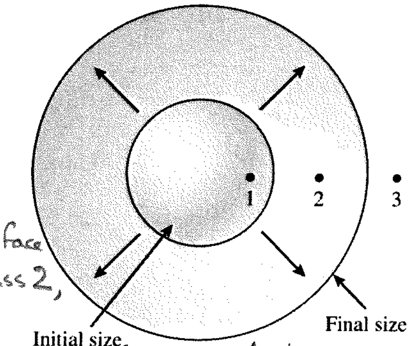
$\Phi_B = -4q/\epsilon_0$

$\Phi_C = 0$

$\Phi_D = 3q/\epsilon_0$

$\Phi_E = 0$

15. A positively charged balloon expands as it is blown up, increasing in size from the initial to final diameters shown. Do the electric fields at points 1, 2, and 3 increase, decrease, or stay the same? Explain your reasoning for each.

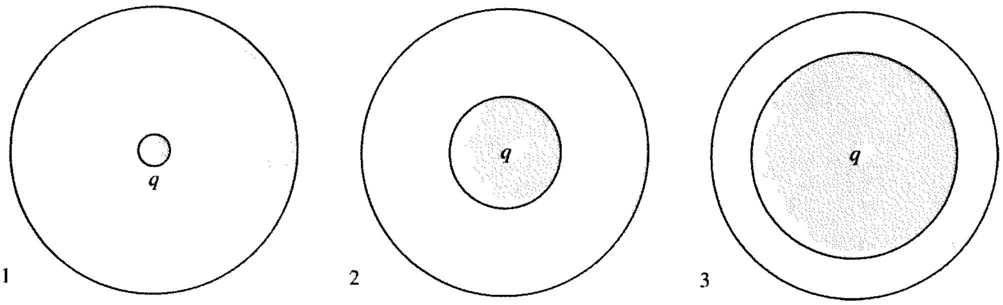


Point 1: Stays the same. A spherical Gaussian surface through 1 never encloses any charge, so the field at 1 is always zero.

Point 2: Decreases. Initially, a spherical Gaussian surface thru 2 gives $\Phi > 0$ and $E_2 \neq 0$. As balloon expands pass 2, the Gaussian surface now gives $\Phi = 0$ and $E_2 = 0$ since no charge is enclosed by the surface; spherical symmetry of external charge.

Point 3: Stays the same. The given charge (say Q) has spherical distribution and is always enclosed by a spherical Gaussian surface through 3. The flux through this surface is always $\Phi = EA = Q/\epsilon_0$, where A is the surface area. So, $E = \frac{Q}{A\epsilon_0}$ = constant (as if entire charge was located at the center).

16. Three charges, all the same charge q, are surrounded by three spheres of equal radii.



a. Rank in order, from largest to smallest, the fluxes Φ_1, Φ_2, and Φ_3 through the spheres.

Order: $\Phi_1 = \Phi_2 = \Phi_3$

Explanation: $\Phi = q/\epsilon_0$ same for all.

The flux is equal to the charge enclosed by the surface divided by ϵ_0.

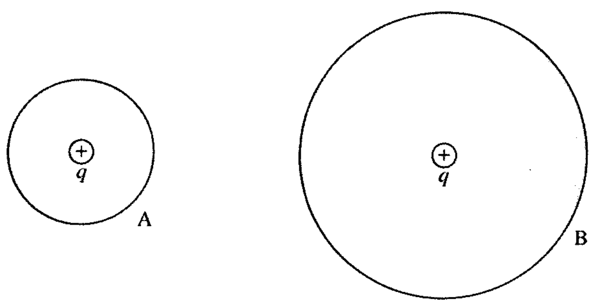
b. Rank in order, from largest to smallest, the electric field strengths E_1, E_2, and E_3 on the surfaces of the spheres.

Order: $E_1 = E_2 = E_3$

Explanation: $E = \dfrac{q}{4\pi r^2 \epsilon_0}$ same q, same r

Field same as that of a point q at the center

17. Two spheres of different diameters surround equal charges. Three students are discussing the situation.



Student 1: The flux through spheres A and B are equal because they enclose equal charges.

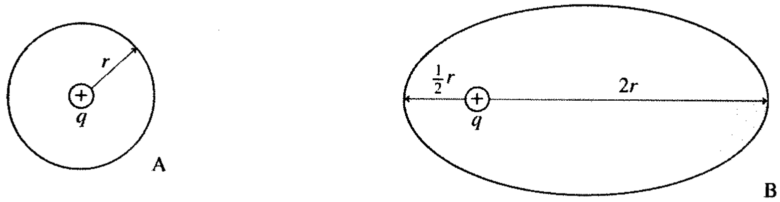
Student 2: But the electric field on sphere B is weaker than the electric field on sphere A. The flux depends on the electric field strength, so the flux through A is larger than the flux through B.

Student 3: I thought we learned that flux was about surface area. Sphere B is larger than sphere A, so I think the flux through B is larger than the flux through A.

Which of these students, if any, do you agree with? Explain.

Student 1. The area increases as r^2 but the electric field strength decreases as $1/r^2$ so the flux is the same through spheres A and B.

18. A sphere and an ellipsoid surround equal charges. Four students are discussing the situation.



Student 1: The fluxes through A and B are equal because the average radius is the same.

Student 2: I agree that the fluxes are equal, but it's because they enclose equal charges.

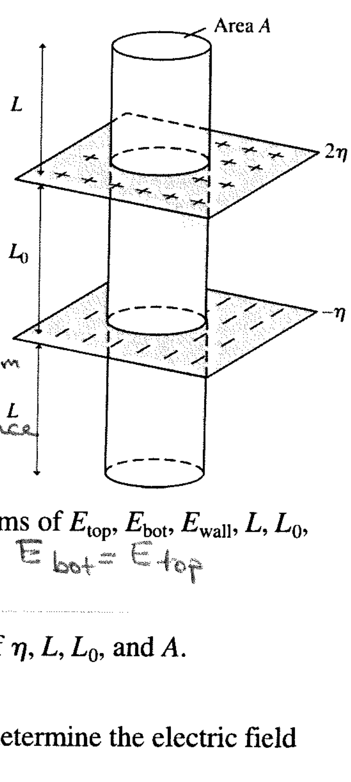
Student 3: The electric field is not perpendicular to the surface for B, and that makes the flux through B less than the flux through A.

Student 4: I don't think that Gauss's law even applies to a situation like B, so we can't compare the fluxes through A and B.

Which of these students, if any, do you agree with? Explain.

Student 2. The flux in both cases is simply the enclosed charge (q) divided by ϵ_0. One could explicitly evaluate the surface integration $\oint \vec{E} \cdot d\vec{A}$, but case B would not be straight forward since the surface does not have spherical symmetry.

19. Two parallel, infinite planes of charge have charge densities
PSS 24.1 2η and $-\eta$. A Gaussian cylinder with cross section area A extends distance L to either side.



a. Is $\vec{E}$ perpendicular or parallel to the Gaussian surface at the:

Top $\perp$ Bottom $\perp$ Wall $\parallel$

b. Is the electric field E_{top} emerging from the top surface stronger than, weaker than, or equal in strength to the field E_{bot} emerging from the bottom? Explain.

Equal in strength, $E_{top} = E_{bot}$. The planes give uniform emerging fields in opposite directions, but the field from the positive plane is twice as strong. So the size difference in these fields gives the strength of the net field emerging from the top and bottom of the cylinder.

c. By inspection, write the electric fluxes through the three surfaces in terms of E_{top}, E_{bot}, E_{wall}, L, L_0, and A. (You may not need all of these.)

or E_{top} since $E_{bot} = E_{top}$

$\Phi_{top} =$ $E_{top} A$ $\Phi_{bot} =$ $E_{bot} A$ $\Phi_{wall} =$ 0

d. How much charge is enclosed within the cylinder? Write Q_{in} in terms of η, L, L_0, and A.

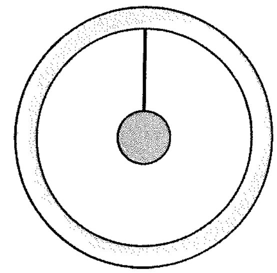
$Q_{in} =$ $(2\eta - \eta)A = \eta A$

e. By combining your answers from parts b, c, and d, use Gauss's law to determine the electric field strength above the top plane. Show your work.

Using the Gaussian cylinder, the total flux through its surface gives $\Phi_e = \Phi_{top} + \Phi_{bot} + \Phi_{wall} = \frac{Q_{in}}{\epsilon_0}$ where $\Phi_{top} = \Phi_{bot} = E_{top} A$ since $E_{top} = E_{bot}$. So, $\Phi_e = E_{top} A + E_{top} A + 0 = \frac{Q_{in}}{\epsilon_0} = \frac{\eta A}{\epsilon_0}$.
Finally, $2E_{top} A = \frac{\eta A}{\epsilon_0}$ gives $\boxed{E_{top} = \frac{\eta}{2\epsilon_0}}$

24.6 Conductors in Electrostatic Equilibrium

20. A small metal sphere hangs by a thread within a larger, hollow conducting sphere. A charged rod is used to transfer positive charge to the outer surface of the hollow sphere.



a. Suppose the thread is an insulator. After the charged rod touches the outer sphere and is removed, are the following surfaces positive, negative, or not charged?

The small sphere: not charged

The inner surface of the hollow sphere: not charged

The outer surface of the hollow sphere: positive

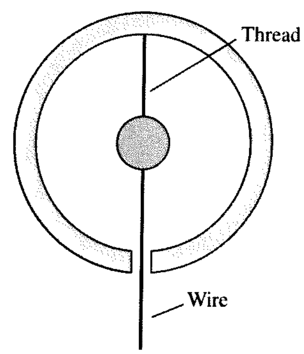
b. Suppose the thread is a conductor. After the charged rod touches the outer sphere and is removed, are the following surfaces positive, negative, or not charged?

The small sphere: not charged

The inner surface of the hollow sphere: not charged

The outer surface of the hollow sphere: positive

21. A small metal sphere hangs by an insulating thread within a larger, hollow conducting sphere. A conducting wire extends from the small sphere through, but not touching, a small hole in the hollow sphere. A charged rod is used to transfer positive charge to the wire. After the charged rod has touched the wire and been removed, are the following surfaces positive, negative, or not charged?

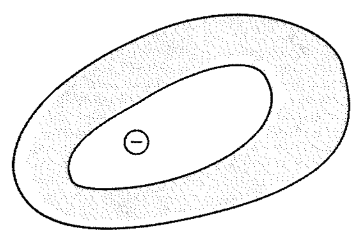


The small sphere: positive

The inner surface of the hollow sphere: negative

The outer surface of the hollow sphere: positive

22. A -10 nC point charge is inside a hole in a conductor. The conductor has no net charge.



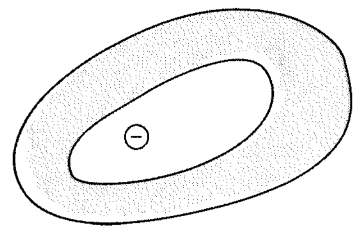
a. What is the total charge on the inside surface of the conductor?

+10 nC

b. What is the total charge on the outside surface of the conductor?

- 10 nC

23. A -10 nC point charge is inside a hole in a conductor. The conductor has a net charge of $+10$ nC.



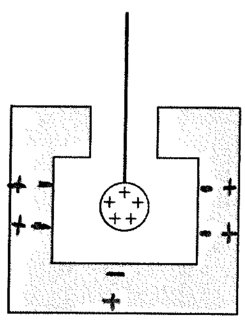
a. What is the total charge on the inside surface of the conductor?

+ 10 nC

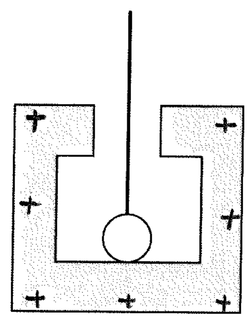
b. What is the total charge on the outside surface of the conductor?

0

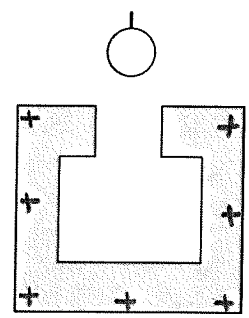
24. An insulating thread is used to lower a positively charged metal ball into a metal container. Initially, the container has no net charge. Use plus and minus signs to show the charge distribution on the ball and the container at the times shown in the figure. (The ball's charge is already shown in the first frame.)



Ball hasn't touched



Ball has touched



Ball has been withdrawn

Negative charges on inside surface
Positive charges on outside surface

Positive charges on outside surface