23 The Electric Field

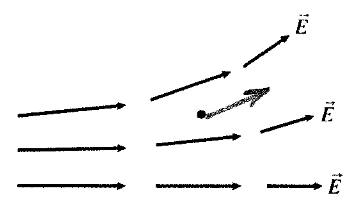
23.1 Electric Field Models

23.2 The Electric Field of Point Charges

1. You've been assigned the task of determining the magnitude and direction of the electric field at a point in space. Give a step-by-step procedure of how you will do so. List any objects you will use, any measurements you will make, and any calculations you will need to perform. Make sure that your measurements do not disturb the charges that are creating the field.

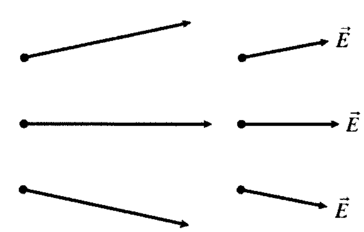
Place a tiny, positive test charge (q) at the point in space and measure the force ($\vec{F}$) on it. Take this measured force and divide it by the test charge to obtain the electric field: $\vec{E} = \vec{F}/q$ where $\vec{F}$ is the force on q.

2. Is there an electric field at the position of the dot? If so, draw the electric field vector on the figure. If not, what would you need to do to create an electric field at this point?



Yes as shown.

3. This is the electric field in a region of space.



a. Explain the information that is portrayed in this diagram.

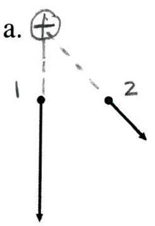
The electric field and net electric force (on a test q) is about twice as strong at the 3 points on the left than at the 3 points on the right. The charge source could be finite, positive and on the left of the 6 points.

b. If field vectors were drawn at the same six points but each was only half as long, would the picture represent the same electric field or a different electric field? Explain.

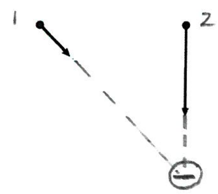
It could represent the same electric field because the length of a given field vector only represents relative magnitude of the electric field at that point.

4. Each figure shows two vectors. Can a point charge create an electric field that looks like this at these two points? If so, draw the charge on the figure. If not, why not?

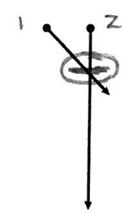
Note: The dots are the points to which the vectors are attached. There are no charges at these points.

a.


Yes. Point 2 is farther away from charge.

b.


Yes. Negative charge source is closer to 2.

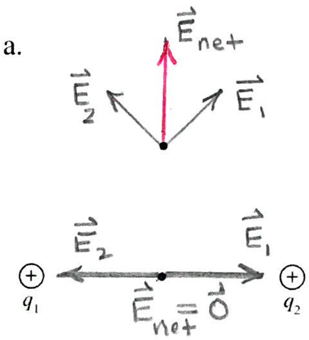
c.


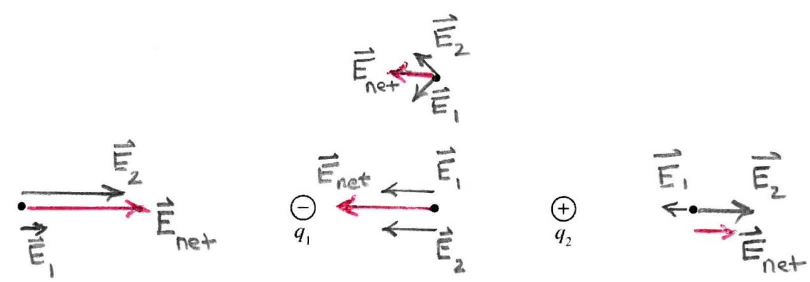
Yes. Negative charge source, closer to 2. Note, vectors are not field lines and may cross.

d.


No. There can only be one net $\vec{E}$ at any given point in space.

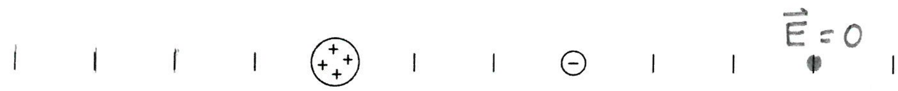
5. At each of the dots, use a **black** pen or pencil to draw and label the electric fields $\vec{E}_1$ and $\vec{E}_2$ due to the two point charges. Make sure that the *relative* lengths of your vectors indicate the strength of each electric field. Then use a **red** pen or pencil to draw and label the net electric field $\vec{E}_{net}$ at each dot.

a.


b.


6. For each of the figures, use dots to mark any point or points (other than infinity) where $\vec{E} = \vec{0}$.

a.


b.


7. Compare the electric field strengths E_1 and E_2 at the two points labeled 1 and 2. For each, is $E_1 > E_2$, is $E_1 = E_2$ or is $E_1 < E_2$?

a.

$E_1 < E_2$

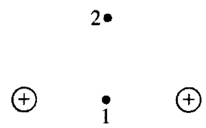
b.

$E_1 > E_2$

c.

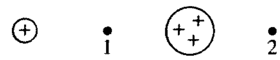
$E_1 = E_2$

d.



$E_1 < E_2$

e.



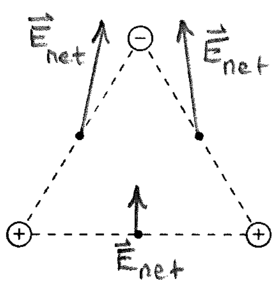
$E_1 < E_2$

f.

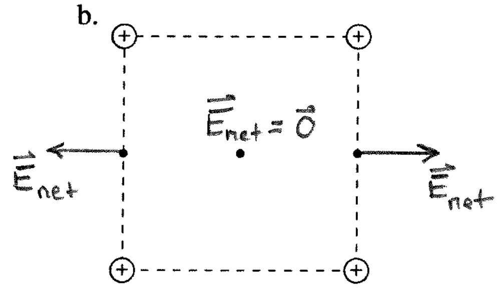
$E_1 > E_2$

8. For each figure, draw and label the net electric field vector $\vec{E}_{net}$ at each of the points marked with a dot or, if appropriate, label the dot $\vec{E}_{net} = \vec{0}$. The lengths of your vectors should indicate the magnitude of $\vec{E}$ at each point.

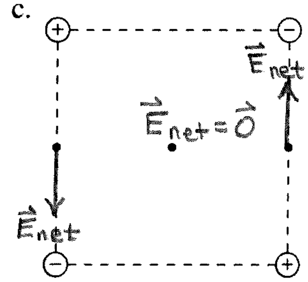
a.



b.

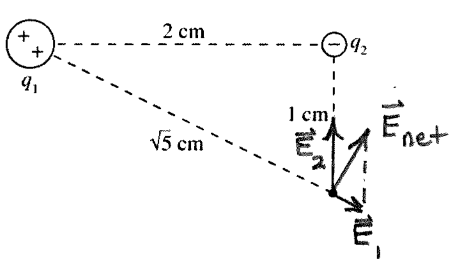


c.



9. At the position of the dot, draw field vectors $\vec{E}_1$ and $\vec{E}_2$ due to q_1 and q_2, and the net electric field $\vec{E}_{net}$. Then, in the blanks, state whether the x- and y-components of $\vec{E}_{net}$ are positive or negative.

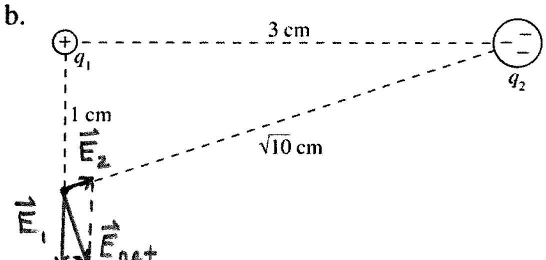
a.



$(E_{net})_x$ positive

$(E_{net})_y$ positive

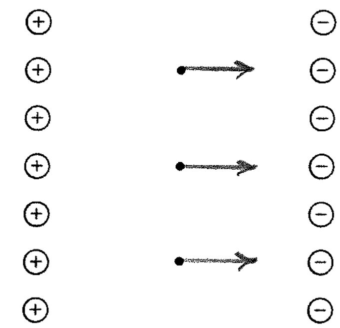
b.



$(E_{net})_x$ positive

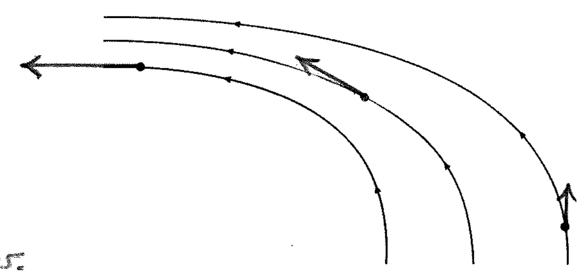
$(E_{net})_y$ negative

10. Draw the net electric field vector at the three points marked with a dot. Hint: Think of the charges as horizontal positive/negative pairs, then use superposition.



Any vertical component from a ⊕ charge is cancelled by the vertical component from ⊖ charge opposite to it.

11. The figure shows the electric field lines in a region of space. Draw the electric field vectors at the three dots. The length of the vector should indicate the relative strength of the electric field at that point.



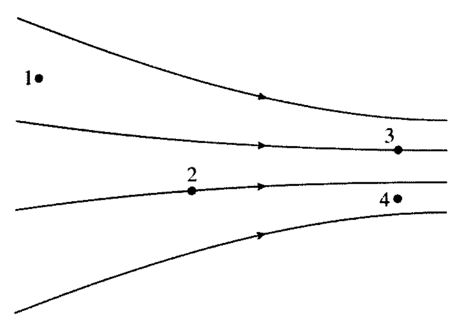
The field is stronger where the field lines are closer together.

12. The figure shows the electric field lines in a region of space. Rank in order, from largest to smallest, the electric field strengths E_1 to E_4 at points 1 to 4.

Order: $E_3 = E_4 > E_2 > E_1$

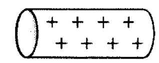
Explanation:

The electric field is stronger in regions where the density of lines is greater, that is, where the lines are closer together.



23.3 The Electric Field of a Continuous Charge Distribution

13. A small segment of wire contains 10 nC of charge.



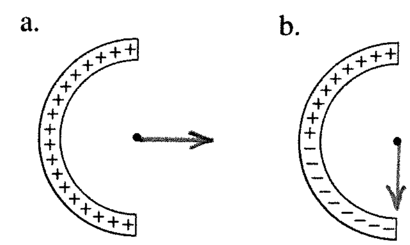
a. The segment is shrunk to one-third of its original length. What is the ratio λ_f/λ_i, where λ_i and λ_f are the initial and final linear charge densities?

$\frac{\lambda_f}{\lambda_i} = \frac{(Q_f/L_f)}{(Q_i/L_i)}$ But since $Q_i = Q_f$, $\frac{\lambda_f}{\lambda_i} = \frac{L_i}{L_f} = 3$

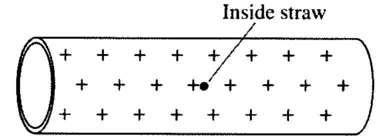
b. Suppose the original segment of wire is stretched to 10 times its original length. How much charge must be added to the wire to keep the linear charge density unchanged?

10 times the original amount of charge would give a constant linear charge density. So the additional charge needed is 9 times the original charge.

14. The figure shows two charged rods bent into a semicircle. For each, draw the electric field vector at the dot at the "center" of the semicircle.



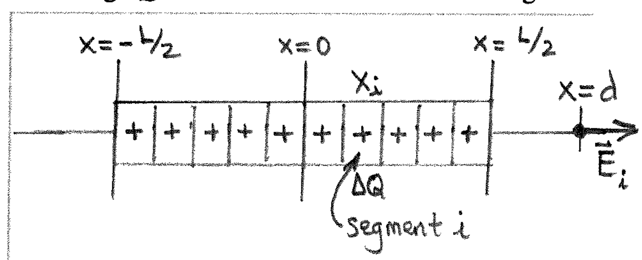
15. A hollow soda straw is uniformly charged. What is the electric field at the center (inside) of the straw? Explain.



Since the straw is uniformly charged, the electric field at the center must be zero.
For example, if $\vec{E}$ was non zero and up that would require more + charge at the bottom giving a nonuniform charge distribution.

16. A thin rod of length L is uniformly charged with total charge Q. What is the electric field strength on the axis of the rod at distance d from its center?
PSS 23.2

a. Begin with a visual representation. Draw a horizontal rod, then divide it into 10 or 12 boxes with a + in each box. Add an x-axis with the rod centered at the origin. Label the ends of the rod $x = -L/2$ and $x = L/2$. Put a dot on the x-axis at some point to the right of the rod; label it $x = d$.



b. Pick one of your + boxes to the right of the origin; label it "segment i," label its position as x_i, and write ΔQ beside it to show the charge in segment i. At the dot, draw the electric field vector due to segment i; label it $\vec{E}_i$.

c. Does $\vec{E}_i$ have an x-component? yes A y-component? no A z-component? no

d. Imagine the electric field $\vec{E}_j$ due to some other segment j.
Is $\vec{E}_j$ the same length as $\vec{E}_i$? no Does $\vec{E}_j$ point the same direction as $\vec{E}_i$? yes

e. The rod's electric field $\vec{E}$ is the sum of all the $\vec{E}_i$. Based on what you've said so far:
Does $\vec{E}$ have an x-component? yes A y-component? no A z-component? no
You should have found that $\vec{E}$ has only one component, requiring only one summation.

f. Using what you know about the electric field of a point charge, write an expression for the electric field component of $\vec{E}_i$—the component you identified in part e—in terms of ΔQ, x_i, d, and various constants.

$$E_{ix} = \frac{1}{4\pi\epsilon_0} \frac{\Delta Q}{(d-x_i)^2}$$

g. Now write an expression of this component of the rod's field $\vec{E}$ as a sum over all i of your answer to part f.

$$E_x = \frac{1}{4\pi\epsilon_0} \sum_i \frac{\Delta Q}{(d-x_i)^2}$$

h. The rod has charge Q in length L. What is the linear charge density? $\lambda =$ Q/L

i. Segment i has width Δx. Based on λ and Δx, the segment has charge $\Delta Q =$ $\lambda \Delta x$

j. Rewrite your answer to part g with this substitution for ΔQ.

$$E_x = \frac{1}{4\pi\epsilon_0} \sum_i \frac{\lambda \Delta x}{(d-x_i)^2}$$

k. Now you're ready to convert the sum to an integral. What are the integration limits?
Lower limit $-L/2$ Upper limit $L/2$

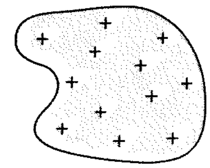
l. Write your expression for the electric field as a definite integral. That means (a) Change Δx to dx, (b) drop the subscript from x_i because x is now a continuous variable, (c) show the integration limits, and (d) take all multiplicative constants outside the integration.

$$E_x = \frac{\lambda}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{dx}{(d-x)^2}$$

We're going to stop here. You've done the physics by figuring out what to integrate. Now it's "just" a calculus problem of carrying out the integration to get a final answer.

23.4 The Electric Fields of Rings, Disks, Planes, and Spheres

17. An irregularly-shaped area of charge has surface charge density η_i. Each dimension (x and y) of the area is reduced by a factor of 3.163.



a. What is the ratio η_f/η_i, where η_f is the final surface charge density?

$$A_f = \frac{A_i}{3.163^2} \quad \text{so} \quad \frac{\eta_f}{\eta_i} = \frac{(Q/A_f)}{(Q/A_i)} = \frac{A_i}{A_f} = 3.163^2 = 10.00$$

b. Compare the final force on a electron very far away to the initial force on the same electron.

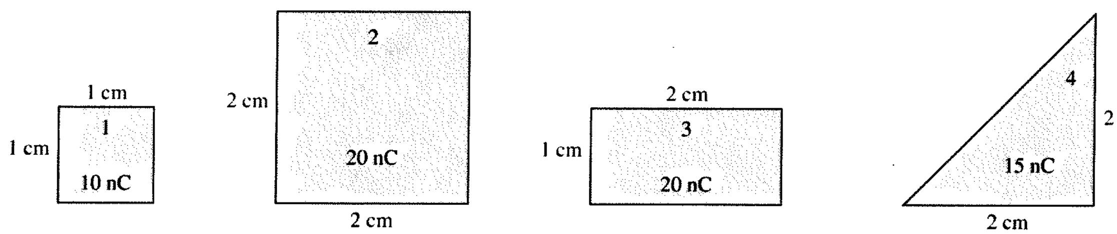
The force will be the same on an electron very far away since any finite charge source will approximate as a point charge.

18. A circular disk has surface charge density 8 nC/cm^2. What will be the surface charge density if the radius of the disk is doubled?

$$\eta_1 = Q/A_1 = Q/\pi r_1^2 = 8\ \text{nC/cm}^2$$

$$\eta_2 = Q/A_2 = Q/\pi r_2^2 = Q/\pi(2r_1)^2 = \frac{1}{4}\frac{Q}{\pi r_1^2} = \frac{1}{4}\eta_1 = 2\ \frac{\text{nC}}{\text{cm}^2}$$

19. Rank in order, from largest to smallest, the surface charge densities η_1 to η_4 of surfaces 1 to 4.



Order: $\eta_1 = \eta_3 > \eta_4 > \eta_2$

Explanation:

$$\eta_1 = Q_1/A_1 = \frac{10\,\text{nC}}{1\text{cm} \times 1\text{cm}} = \frac{10\,\text{nC}}{\text{cm}^2}, \quad \eta_2 = \frac{20\,\text{nC}}{2\text{cm} \times 2\text{cm}} = 5\ \frac{\text{nC}}{\text{cm}^2}$$

$$\eta_3 = \frac{20\,\text{nC}}{1\text{cm} \times 2\text{cm}} = 10\ \frac{\text{nC}}{\text{cm}^2}, \quad \eta_4 = \frac{15\,\text{nC}}{\frac{1}{2}(2\text{cm})(2\text{cm})} = 7.5\ \frac{\text{nC}}{\text{cm}^2}$$

20. A sphere of radius R has charge Q. What happens to the electric field strength at $r = 2R$ if:

a. The quantity of charge is halved?

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$ If Q is halved, then E is also halved.

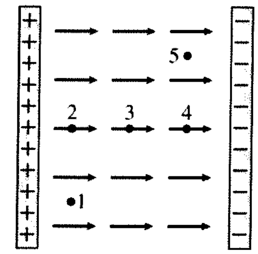
b. The radius of the sphere is halved?

$R_i = R$, $R_f = R/2$ but $E = \dfrac{Q}{4\pi\epsilon_0 r^2} = E_i = E_f$

Provided $r = 2R$ remains the same and Q remains the same.

23.5 The Parallel-Plate Capacitor

21. Rank in order, from largest to smallest, the electric field strengths E_1 to E_5 at each of these points.



Order: $E_1 = E_2 = E_3 = E_4 = E_5$

Explanation:

The electric field between parallel plates is uniform. The electric field vectors shown all have the same length and direction.

22. A parallel-plate capacitor is constructed of two square plates, size $L \times L$, separated by distance d. The plates are given charge $\pm Q$. What is the ratio E_f/E_i of the final electric field strength E_f to the initial electric field strength E_i if:

a. Q is doubled?

Given $Q_f = 2Q$ and $A_f = A_i = A$ (constant)

$$\frac{E_f}{E_i} = \frac{\eta_f/\epsilon_0}{\eta_i/\epsilon_0} = \frac{\eta_f}{\eta_i} = \frac{Q_f/A}{Q/A} = \frac{2Q/A}{Q/A} = 2$$

b. L is doubled?

Given $Q_f = Q_i = Q$ but $A_f = 2L \times 2L = 4L^2 = 4A_i = 4A$

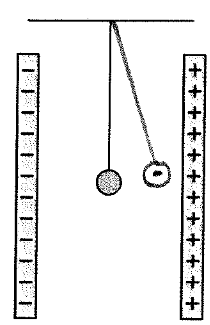
$$\frac{E_f}{E_i} = \frac{\eta_f/\epsilon_0}{\eta_i/\epsilon_0} = \frac{Q/A_f}{Q/A} = \frac{Q/4A}{Q/A} = \frac{1}{4}$$

c. d is doubled?

E does not depend on d.

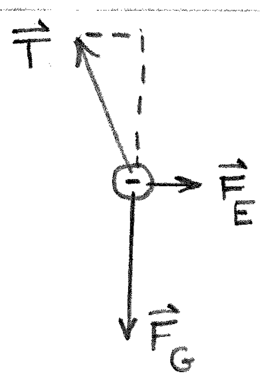
$$\frac{E_f}{E_i} = 1$$

23. A ball hangs from a thread between two vertical capacitor plates. Initially, the ball hangs straight down. The capacitor plates are charged as shown, then the ball is given a small negative charge. The ball moves to one side, but not enough to touch a capacitor plate.



a. Draw the ball and thread in the ball's new equilibrium position.

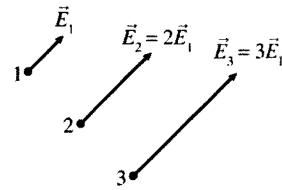
b. In the space below, draw a free-body diagram of the ball when in its new position.



23.6 Motion of a Charged Particle in an Electric Field

23.7 Motion of a Dipole in an Electric Field

24. A small positive charge q experiences force $\vec{F}_1$ when placed at point 1. In terms of $\vec{F}_1$:



   a. What is the force on charge q at point 3? $3\vec{F}_1$

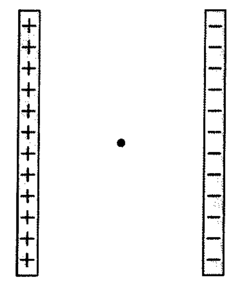
   b. What is the force on a charge $3q$ at point 1? $3\vec{F}_1$

   c. What is the force on a charge $2q$ at point 2? $4\vec{F}_1$

   d. What is the force on a charge $-2q$ at point 2? $-4\vec{F}_1$

25. A small object is released from rest in the center of the capacitor. For each situation, does the object move to the right, to the left, or remain in place? If it moves, does it accelerate or move at constant speed?

Ignore gravity on object



   a. Positive object.

   Accelerates to the right.

   b. Negative object.

   Accelerates to the left.

   c. Neutral object.

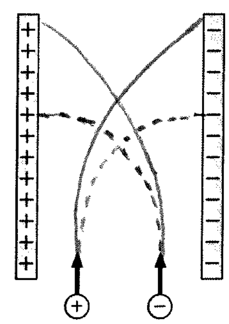
   Remains in place.

26. Positively and negatively charged objects, with equal masses and equal quantities of charge, enter the capacitor in the directions shown.

   a. Use solid lines to draw their trajectories on the figure if their initial velocities are fast.

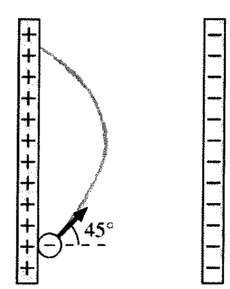
   b. Use dashed lines to draw their trajectories on the figure if their initial velocities are slow.

   Ignore gravity on objects



27. An electron is launched from the positive plate at a 45° angle. It does not have sufficient speed to make it to the negative plate. Draw its trajectory on the figure.

   Parabolic path, landing at 45°



28. First a proton, later an electron are released from rest in the center of a capacitor.

  a. Compare the forces on the two charges. Are they equal, or is one larger? Explain.

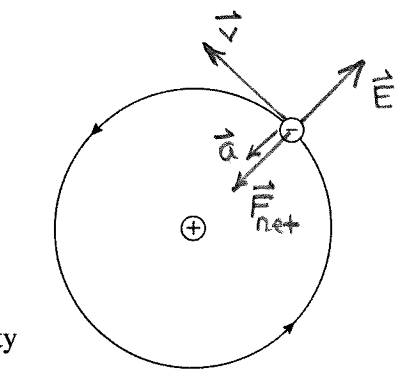
The size of the force on each charge is the same ($F_E = qE$ where $q = +e$ or $q = -e$). The forces point in opposite directions.

  b. Compare the accelerations of the two charges. Are they equal, or is one larger? Explain.

The electron has larger acceleration since it has smaller mass. $a = \frac{F_{net}}{m} = \frac{F_E}{m}$

Note: $F_E \gg F_G$ on each particle.

29. The figure shows an electron orbiting a proton in a hydrogen atom.



  a. What force or forces act on the electron?

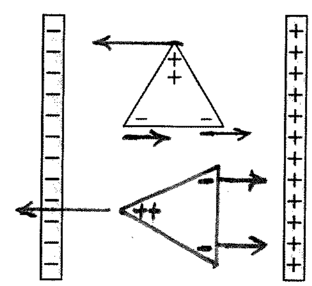
The electric force acts on the electron. (The force of gravity is negligible.)

  b. Draw and label the following vectors on the figure: the electron's velocity $\vec{v}$ and acceleration $\vec{a}$, the net force $\vec{F}_{net}$ on the electron, and the electric field $\vec{E}$ at the position of the electron.

30. Does a charged particle always move in the direction of the electric field? If so, explain why. If not, give an example that is otherwise.

No. At any given instant, the velocity (which gives the direction of motion) of a charged particle is independent of $\vec{E}$.
In problem 27, the initial velocity is 45° from $\vec{E}$.
In problem 29, the velocity is perpendicular to $\vec{E}$.

31. Three charges are placed at the corners of a triangle. The ++ charge has twice the quantity of charge of the two − charges; the net charge is zero.



  a. Draw the force vectors on each of the charges.

  b. Is the triangle in equilibrium? No. If not, draw the equilibrium orientation directly beneath the triangle that is shown.

  c. Once in the equilibrium orientation, will the triangle move to the right, move to the left, rotate steadily, or be at rest? Explain.

In equilibrium (as drawn) the triangle will remain in place since both the net force and the net torque are zero. (Assume the force of gravity is negligible.)