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Modern Optics and Matter Waves

25.1 Spectroscopy: Unlocking the Structure of Atoms

25.2 X-Ray Diffraction

25.3 Photons

1. The figure shows the spectrum of a gas discharge tube.



What color would the discharge appear to your eye? Explain.

Blue, because most of the emitted wavelengths fall in the 430-510 nm range.

2. The first-order x-ray diffraction of monochromatic x rays from a crystal occurs at angle θ_1 . The crystal is then compressed, causing a slight reduction in its volume. Does θ_1 increase, decrease, or stay the same? Explain.

θ_1 decreases. As the crystal is compressed, the spacing 'd' between planes of atoms decreases. The Bragg condition is $m\lambda = 2d \cos \theta_m$ so as d decreases, $\cos \theta_m$ must increase. But $\cos \theta$ increases as θ decreases.

3. Three laser beams have wavelengths $\lambda_1 = 400$ nm, $\lambda_2 = 600$ nm, and $\lambda_3 = 800$ nm. The power of each laser beam is 1 W.
 - a. Rank in order, from largest to smallest, the photon energies E_1 , E_2 , and E_3 in these three laser beams.

Order: $E_1 > E_2 > E_3$

Explanation:

The energy per photon depends only on the frequency so $E = hf = hc/\lambda$. The smaller wavelengths correspond to higher frequencies.

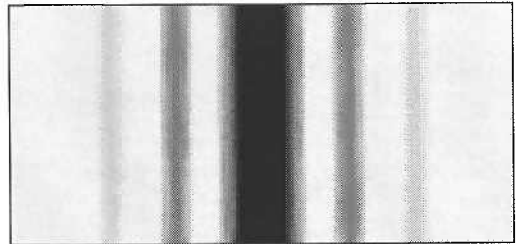
- b. Rank in order, from largest to smallest, the number of photons per second N_1 , N_2 , and N_3 delivered by the three laser beams.

Order: $N_3 > N_2 > N_1$

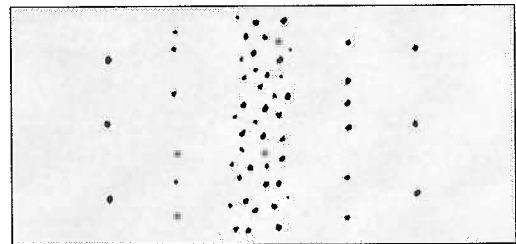
Explanation:

Because the powers are equal, there must be more photons when the energy per photon is less.

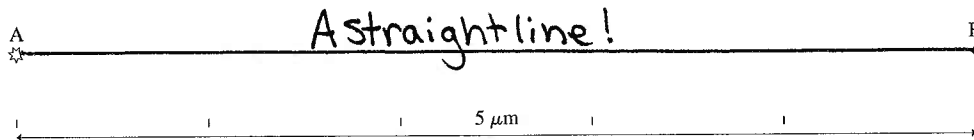
4. The top figure is the *negative* of the photograph of a single-slit diffraction pattern. That is, the darkest areas in the figure were the brightest areas on the screen. This photo was made with an extremely large number of photons.



Suppose the slit is illuminated by an extremely weak light source, so weak that only 1 photon passes through the slit every second. Data are collected for 60 seconds. Draw 60 dots on the empty screen to show how you think the screen will look after 60 photons have been detected.

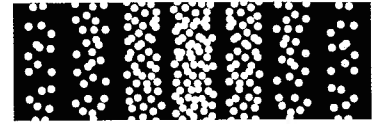


5. A light source at point A emits light with a wavelength of $1.0 \mu\text{m}$. One photon of light is detected at point B, $5.0 \mu\text{m}$ away from A. On the figure, draw the trajectory that the photon follows from A to B.



25.4 Matter Waves

6. The figure is a simulation of the electrons detected behind a very narrow double slit. Each bright dot represents one electron. How will this pattern change if the following experimental conditions are changed? Possible changes you should consider include the number of dots and the spacing, width, and positions of the fringes.



- a. The electron-beam intensity is increased.

The number of dots will increase, but will still appear in bands of the same width.

- b. The electron speed is reduced.

When the speed is reduced, the wavelength increases leading to wider, more spread-out fringes with the same number of dots in each.

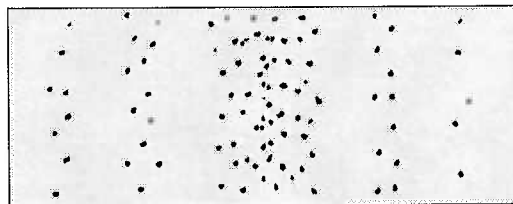
- c. The electrons are replaced by positrons with the same speed. Positrons are antimatter particles that are identical to electrons except that they have a positive charge.

The fringe pattern is unchanged.

- d. One slit is closed.

The pattern becomes a single slit diffraction pattern. The two-slit interference pattern is eliminated.

7. Very slow neutrons pass through a single, very narrow slit. Use 50 or 60 dots to show how the neutron intensity will appear on a neutron-detector screen behind the slit.



8. To have the best resolution, should an electron microscope use very fast electrons or very slow electrons? Explain.

Fast electrons will have a shorter wavelength leading to less diffraction spreading and better resolution.

25.5 Energy Is Quantized

9. a. For the first few allowed energies of a particle in a box to be large, should the box be very big or very small? Explain.

The box should be very small. $E_n = n^2 \frac{h^2}{8mL^2}$, where L is the length of the box. For a given n , small L leads to large energy.

- b. Which is likely to have larger values for the first few allowed energies: an atom in a molecule, an electron in an atom, or a proton in a nucleus? Explain.

A proton in a nucleus, then an electron in an atom, and finally, an atom in a molecule. As the size scale increases, the allowed energies become smaller. Though the proton's mass is 1800× greater than the electron's, the nuclear length scale is much smaller and a more significant factor (L^2).

10. The smallest allowed energy of a particle in a box is 2.0×10^{-20} J. What will be the smallest allowed energy if the length of the box is doubled and the particle's mass is halved?

The smallest allowed energy is halved. 1.0×10^{-20} J

$$E = \frac{h^2}{8m_0L_0^2} \quad E' = \frac{h^2}{8(\frac{m_0}{2})(2L_0)^2} = \frac{E}{2}$$