15 Oscillations

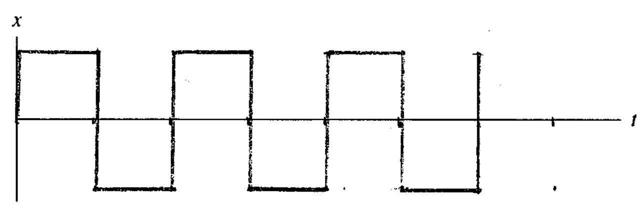
15.1 Simple Harmonic Motion

1. Give three examples of *oscillatory* motion. (Note that circular motion is not the same as oscillatory motion.)

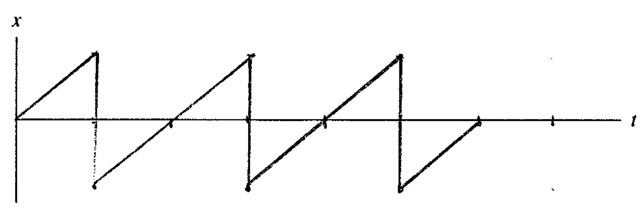
 1. A pendulum swinging.
 2. A rocking chair.
 3. A bouncing ball.
 4. A vibrating tuning fork.
 5. A water wave. Water molecules oscillate.
 6. A beating heart.
 7. AC electric current and voltage.
 8. A mass oscillating as it hangs from a spring.

2. On the axes below, sketch three cycles of the displacement-versus-time graph for:

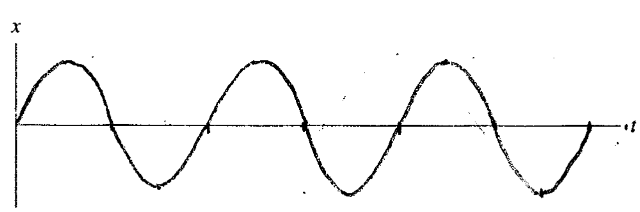
 a. A particle undergoing symmetric periodic motion that is *not* SHM.



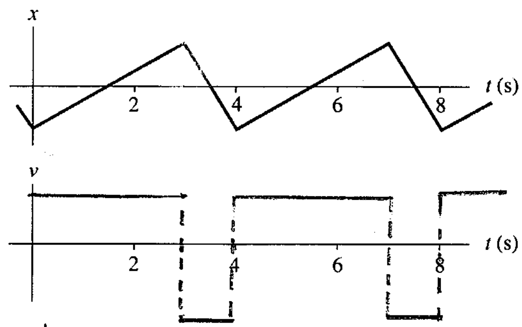
 b. A particle undergoing asymmetric periodic motion.



 c. A particle undergoing simple harmonic motion.



3. Consider the particle whose motion is represented by the x-versus-t graph below.



a. Is this periodic motion? Yes

b. Is this motion SHM? No

c. What is the period? 4 sec

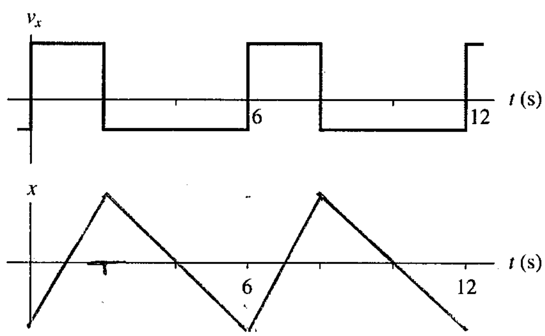
d. What is the frequency? $f = \frac{1}{T} = 0.25\,Hz$

e. You learned in Chapter 2 to relate velocity graphs to position graphs. Use that knowledge to draw the particle's velocity-versus-time graph on the axes provided.

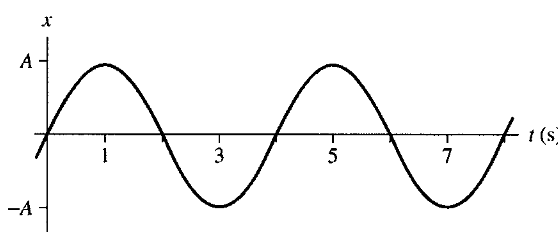
4. Shown below is the velocity-versus-time graph of a particle.

a. What is the period of the motion? 6 s

b. Draw the particle's position-versus-time graph for an oscillation around $x = 0$.



5. The figure shows the position-versus-time graph of a particle in SHM.



a. At what times is the particle moving to the right at maximum speed?

0, 4, 8 s

b. At what times is the particle moving to the left at maximum speed?

2, 6 s

c. At what times is the particle instantaneously at rest?

1, 3, 5, 7 s

15.2 SHM and Circular Motion

6. A particle goes around a circle 5 times at constant speed, taking a total of 2.5 seconds.
   a. Through what angle *in degrees* has the particle moved? $5 \times 360° = 1800°$
   b. Through what angle *in radians* has the particle moved? $5 \times 2\pi = 10\pi$ (rads)
   c. What is the particle's frequency f?

   $$\frac{5 \text{ cycles}}{2.5 \text{ s}} = 2.0 \text{ Hz}$$

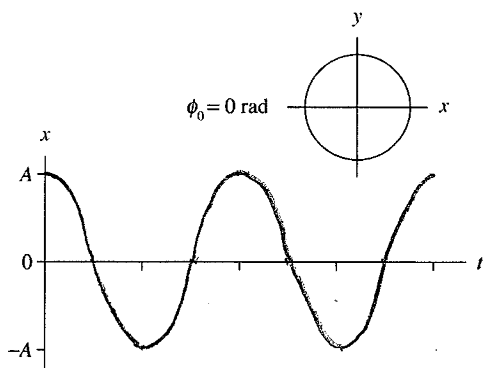
   d. Use your answer to part b to determine the particle's angular frequency ω.

   $\omega = 2\pi f = 4\pi$ Hz or $12.6/s$ or 12.6 rad/s

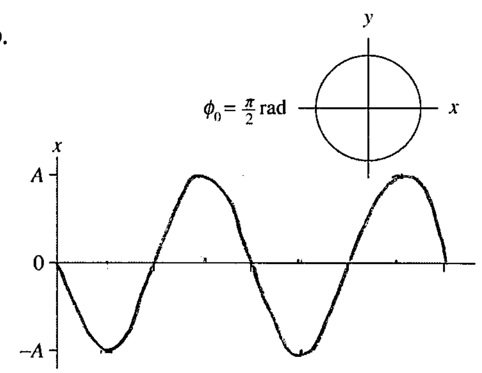
   e. Does ω (in rad/s) $= 2\pi f$(in Hz)? Yes

7. A particle moves counterclockwise around a circle at constant speed. For each of the phase constants given below:
   - Show with a dot *on the circle* the particle's starting position.
   - Sketch two cycles of the particle's x-versus-t graph.

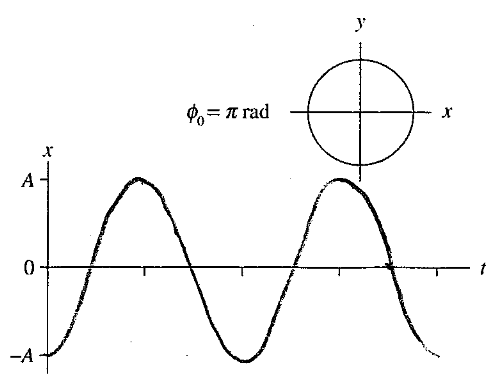
a.



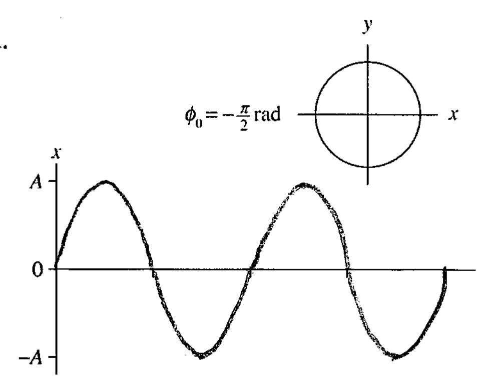
b.



c.

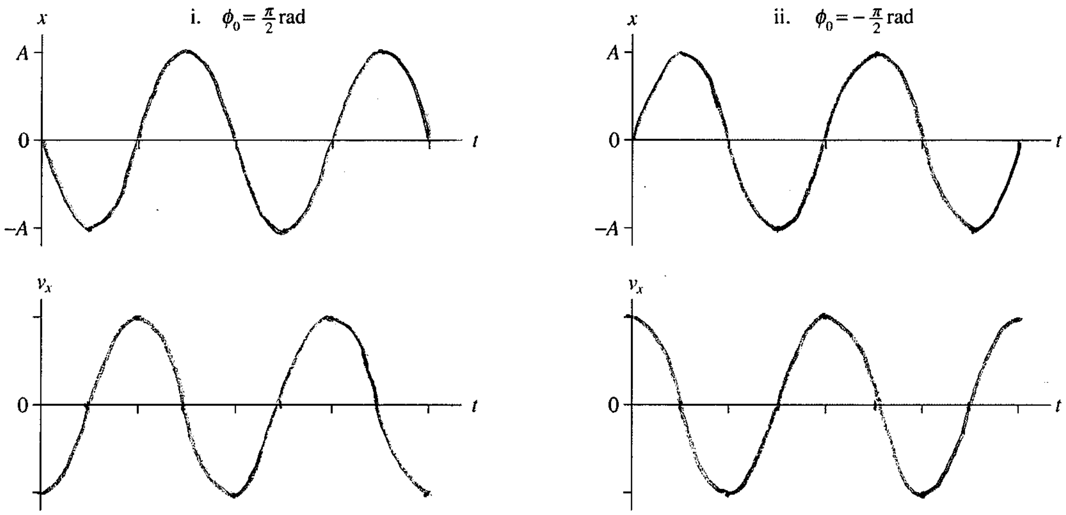


d.

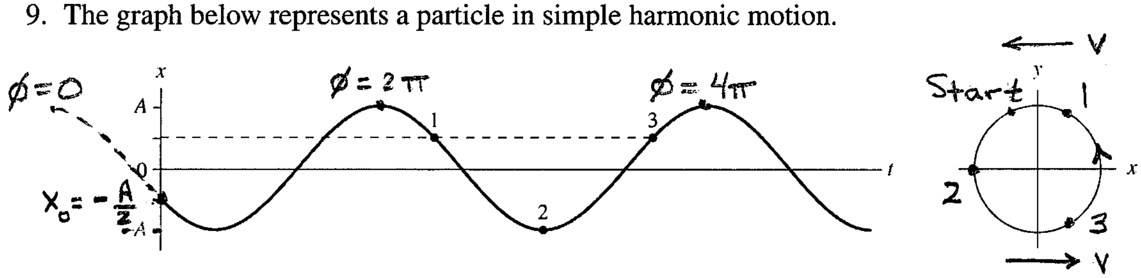


8. a. On the top set of axes below, sketch two cycles of the x-versus-t graphs for a particle in simple harmonic motion with phase constants i) $\phi_0 = \pi/2$ rad and ii) $\phi_0 = -\pi/2$ rad.

   b. Use the bottom set of axes to sketch velocity-versus-time graphs for the particles. Make sure each velocity graph aligns vertically with the correct points on the x-versus-t graph.



9. The graph below represents a particle in simple harmonic motion.



   a. What is the phase constant ϕ_0? Explain how you determined it.

   At start, $X_0 = -\frac{A}{2} = A\cos(\phi_0)$ gives $\phi_0 = \pm \frac{2\pi}{3}$
   But ϕ_0 must be unique. Look at sign of slope at $t = 0$ to see $V_0 < 0$ giving $\boxed{\phi_0 = +\frac{2\pi}{3}}$ uniquely since the circular-motion particle starts in the 2nd quadrant where $V_0 < 0$ (and not the 3rd quadrant where $V > 0$).

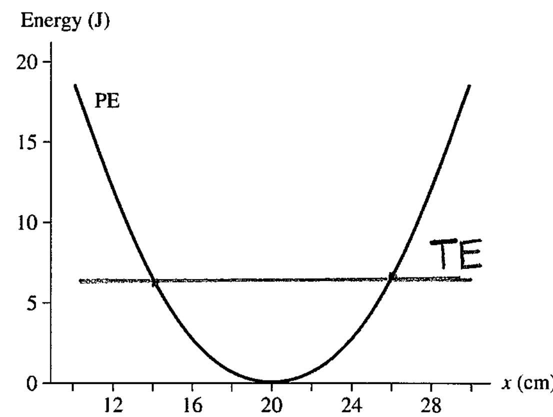
   b. What is the phase of the particle at each of the three numbered points on the graph?

   Phase at 1: $2\pi + \frac{\pi}{3} = \frac{7\pi}{3}$ Phase at 2: $2\pi + \pi = 3\pi$ Phase at 3: $3\pi + \frac{2\pi}{3} = \frac{11\pi}{3}$

   c. Place dots on the circle above to show the position of a circular-motion particle at the times corresponding to points 1, 2, and 3. Label each dot with the appropriate number.

15.3 Energy in SHM

10. The figure shows the potential-energy diagram of a particle oscillating on a spring.



a. What is the spring's equilibrium length?

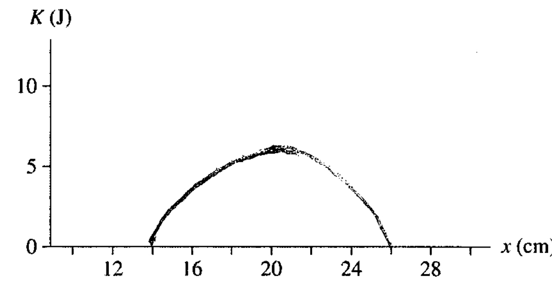
20 cm

b. The particle's turning points are at 14 cm and 26 cm. Draw the total energy line and label it TE.

c. What is the particle's maximum kinetic energy?

About 6.5 J

d. Draw a graph of the particle's kinetic energy as a function of position.



e. What will be the turning points if the particle's total energy is doubled?

12 cm, 28 cm

11. A block oscillating on a spring has an amplitude of 20 cm. What will be the block's amplitude if its total energy is tripled? Explain.

$E = \frac{1}{2}kA^2$. Let $E_1 = \frac{1}{2}kA_1^2$ where $A_1 = 20\text{cm}$.
Suppose $E_2 = 3E_1$ using the same spring, then

$$3 = \frac{E_2}{E_1} = \frac{\frac{1}{2}kA_2^2}{\frac{1}{2}kA_1^2} = \frac{A_2^2}{A_1^2} \quad \text{so} \quad A_2 = \sqrt{3}\,A_1 = \sqrt{3}\,(20\text{cm})$$

$$\boxed{A_2 = 34.6\text{cm}}$$

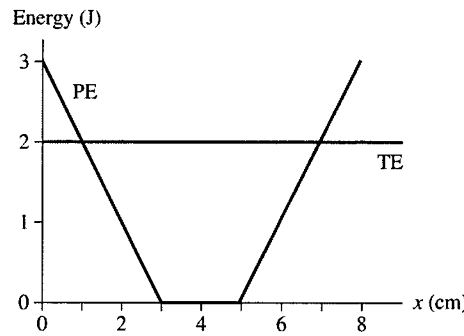
12. A block oscillating on a spring has a maximum speed of 20 cm/s. What will be the block's maximum speed if its total energy is tripled? Explain.

$E = \frac{1}{2}mv_{max}^2$. Let $E_1 = \frac{1}{2}mv_{1\,max}^2$ where $v_{1\,max} = \frac{20\text{cm}}{s}$.
Suppose $E_2 = 3E_1$ using the same block mass on a spring, then

$$3 = \frac{E_2}{E_1} = \frac{\frac{1}{2}mv_{2\,max}^2}{\frac{1}{2}mv_{1\,max}^2} = \frac{v_{2\,max}^2}{v_{1\,max}^2} \quad \text{so} \quad v_{2\,max} = \sqrt{3}\,v_{1\,max}$$

$$\boxed{v_{2\,max} = 34.6\,\text{cm/s}}$$

13. The figure shows the potential energy diagram of a particle.



a. Is the particle's motion periodic? How can you tell?

Yes, the particle oscillates between its turning points ($x = 1$ cm and $x = 7$ cm).

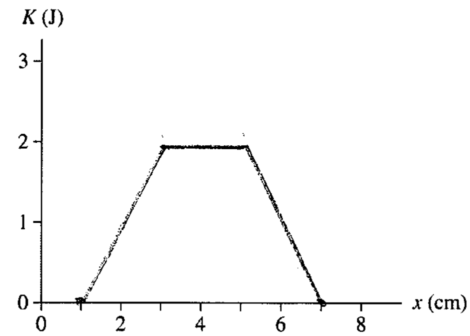
b. Is the particle's motion simple harmonic motion? How can you tell?

No. The PE curve is not quadratic.

c. What is the amplitude of the motion?

Amplitude = 3 cm as seen by taking the difference between a turning point location and equilibrium.

d. Draw a graph of the particle's kinetic energy as a function of position.



14. Equation 15.25 in the textbook states that $\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$. What does this mean? Write a couple of sentences explaining how to interpret this equation.

An object in SHM has a total mechanical energy that is the sum of its elastic potential energy and its kinectic energy. But at the turning points, that object has no kinetic energy so the total energy (which is conserved throughout) is 100% potential energy. Now, when the object is at equilibrium, the total energy is 100% kinetic energy. Since total energy is conserved throughout the motion, the total energy at a turning point must be the same as the total energy at equilibrium (giving equation 15.25 in the textbook)

15.4 The Dynamics of SHM

15.5 Vertical Oscillations

15. A block oscillating on a spring has period $T = 4$ s.
    a. What is the period if the block's mass is halved? Explain.
    Note: You do not know values for either m or k. Do *not* assume any particular values for them. The required analysis involves thinking about ratios.

Period is 2.83 s when mass is halved.

$T = 2\pi\sqrt{\frac{m}{k}} = 4\,s$ as given.

$T' = 2\pi\sqrt{\frac{m/2}{k}} = \frac{1}{\sqrt{2}}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{1}{\sqrt{2}}(4s) = 2.83\,s.$

b. What is the period if the value of the spring constant is quadrupled?

Period is 2 s when spring constant is quadrupled.

$T = 2\pi\sqrt{\frac{m}{k}} = 4\,s$ as given.

$T' = 2\pi\sqrt{\frac{m}{4k}} = \frac{1}{2}\left(2\pi\sqrt{\frac{m}{k}}\right) = \frac{1}{2}(4s) = 2s$

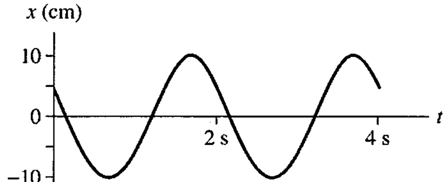
c. What is the period if the oscillation amplitude is doubled while m and k are unchanged?

Period is 4 s (unchanged) since T is independent of amplitude.

16. For graphs a and b, determine:
    - The angular frequency ω.
    - The oscillation amplitude A.
    - The phase constant ϕ_0.

Note: Graphs a and b are independent. Graph b is *not* the velocity graph of a.

a.

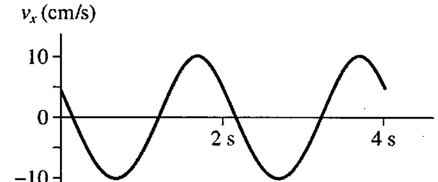


$\omega = \pi \frac{rad}{s} \approx 3.14 \frac{rad}{s} \left(\omega = \frac{2\pi}{T}\right)$

$A = 10\ cm$

$\phi_0 = \frac{\pi}{3}$ rad or $60°$

b.

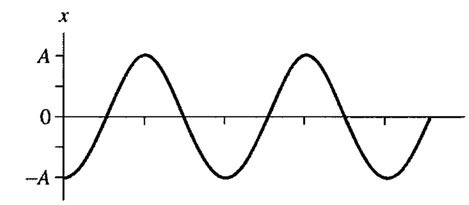


$\omega = \pi \frac{rad}{s} \approx 3.14 \frac{rad}{s} \left(\omega = \frac{2\pi}{T}\right)$

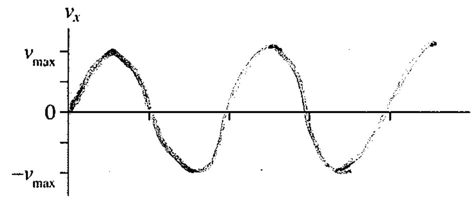
$A = \frac{10}{\pi} cm \approx 3.18\ cm\ (v_{max} = \omega A)$

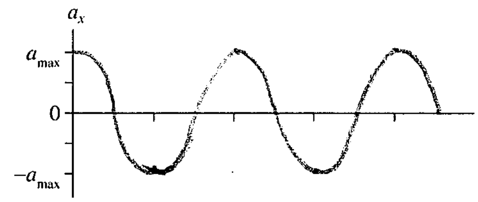
$\phi_0 = -\pi/6$ rad or $-30°$

17. The graph on the right is the position-versus-time graph for a simple harmonic oscillator.



a. Draw the v_x-versus-t and a_x-versus-t graphs.





b. When x is greater than zero, is a_x ever greater than zero? If so, at which points in the cycle?

No.

c. When x is less than zero, is a_x ever less than zero? If so, at which points in the cycle?

No.

d. Can you make a general conclusion about the relationship between the sign of x and the sign of a_x?

The signs of x and a are opposite.

e. When x is greater than zero, is v_x ever greater than zero? If so, how is the oscillator moving at those times?

Yes. When x and V_x have the same sign, the oscillator is slowing down as it approaches a turning point.

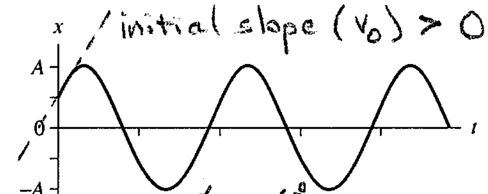
18. For the oscillation shown on the left below:

a. What is the phase constant ϕ_0? $\phi_0 = -60^\circ$ since $\cos\phi_0 = \frac{1}{2}$ and $-\sin\phi_0 > 0$

b. Draw the corresponding v_x-versus-t graph on the axes below the x-versus-t graph.

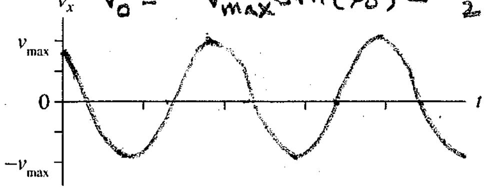
c. On the axes on the right, sketch two cycles of the x-versus-t and the v_x-versus-t graphs if the value of ϕ_0 found in part a is replaced by its negative, $-\phi_0$.

initial slope $(V_0) > 0$

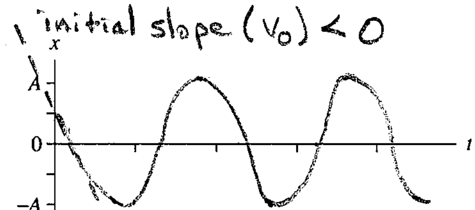


$\phi_0 = -60^\circ$

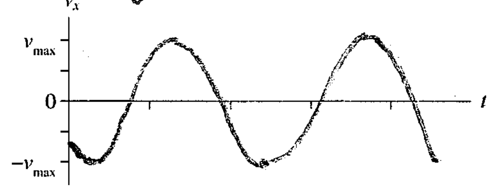
$V_0 = -V_{max}\sin(\phi_0) = \frac{\sqrt{3}}{2} V_{max}$



initial slope $(V_0) < 0$



$V_0 = -V_{max}\sin(60^\circ) = -\frac{\sqrt{3}}{2} V_{max}$

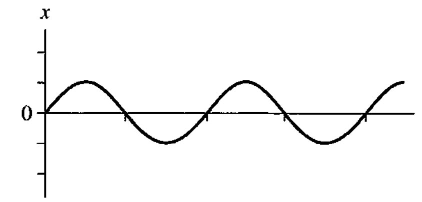


d. Describe *physically* what is the same and what is different about the initial conditions for two oscillators having "equal but opposite" phase constants ϕ_0 and $-\phi_0$.

Both oscillators have $X_0 = \frac{A}{2}$. When $\phi_0 = -60^\circ$ the initial velocity is positive $(V_0 > 0)$ as the oscillator is slowing down, but when initial phase is $+60^\circ$ the initial velocity is negative $(V_0 < 0)$ as the oscillator is speeding up.

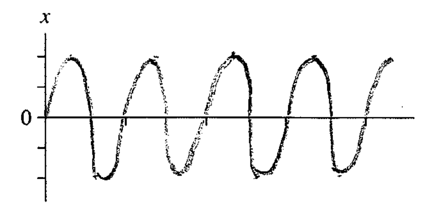
19. The top graph shows the position versus time for a mass oscillating on a spring. On the axes below, sketch the position-versus-time graph for this block for the following situations:

    Note: The changes described in each part refer back to the original oscillation, not to the oscillation of the previous part of the question. Assume that all other parameters remain constant. Use the same horizontal and vertical scales as the original oscillation graph.



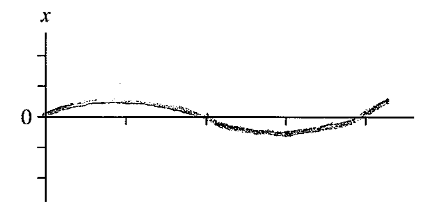
a. The amplitude and the frequency are doubled.

Period $T = \frac{1}{f}$

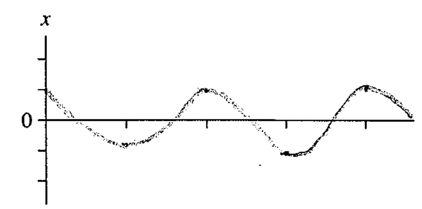


b. The amplitude is halved and the mass is quadrupled.

Period $T \propto \sqrt{m}$



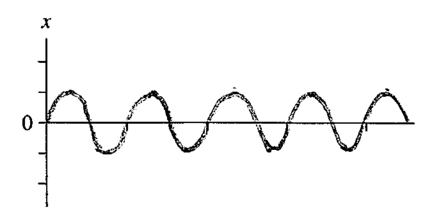
c. The phase constant is increased by $\pi/2$ rad.



d. The maximum speed is doubled while the amplitude remains constant.

$V_{max} = \omega A = (2\pi f) A$

so $T \propto \frac{1}{V_{max}}$



15.6 The Pendulum

20. A pendulum on planet X, where the value of g is unknown, oscillates with a period of 2 seconds. What is the period of this pendulum if:

a. Its mass is tripled?

Note: You do not know the values of m, L, or g, so do not assume any specific values.

$T = \sqrt{\frac{L}{g}}$ is independent of mass, so $\boxed{T = 2s}$

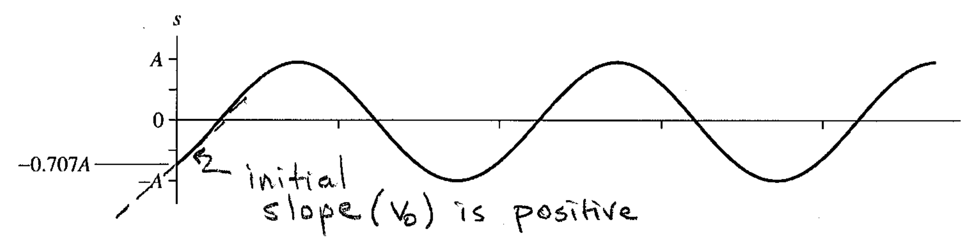
b. Its length is tripled?

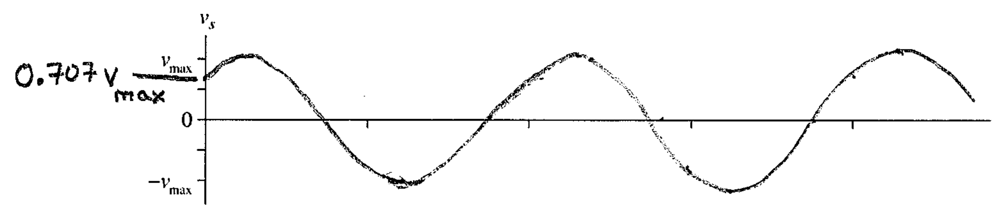
$\frac{T_2}{T_1} = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{3L_1}{L_1}} = \sqrt{3}$ or $T_2 = \sqrt{3}\,T_1$ so $\boxed{T \approx 3.5s}$

c. Its oscillation amplitude is tripled?

The period is independent of amplitude for small angles.
$\boxed{T \approx 2s}$

21. The graph shows the displacement s versus time for an oscillating pendulum.



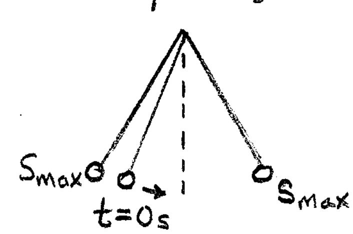


a. Draw the pendulum's velocity-versus-time graph.
b. What is the value of the phase constant ϕ_0?

$\phi_0 = -\frac{3\pi}{4}$ or $-135°$. Use S_0 and V_0 to find ϕ_0.
$S_0 = -0.707A = A\cos\phi_0$ gives $\phi_0 = \pm\frac{3\pi}{4}$ but also requiring
$V_0 = +0.707\,V_{max} = -V_{max}\sin\phi_0$ gives $\phi_0 = -\frac{3\pi}{4}$ uniquely.

c. In the space at the right, draw a *picture* of the pendulum that shows (and labels!)

- The extremes of its motion.
- Its position at $t = 0$ s.
- Its direction of motion (using an arrow) at $t = 0$ s.

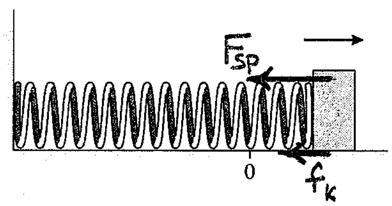


15.7 Damped Oscillations

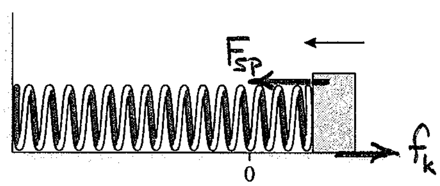
22. Can the following be reasonably modeled as SHM? Answer Yes or No.
    a. A perfectly elastic ball bouncing up and down to the same height. No
    b. A marble rolling in the bottom of a bowl. Yes, with damping
    c. A plastic ruler clamped at one end and "plucked" at the other. Yes, with damping
    d. A heavy mass hanging from a rope that is twisting back and forth. Yes, with damping

23. If the damping constant b of an oscillator is increased,
    a. Is the medium more resistive or less resistive? more resistive
    b. Do the oscillations damp out more quickly or less quickly? more quickly
    c. Is the time constant τ increased or decreased? decreased

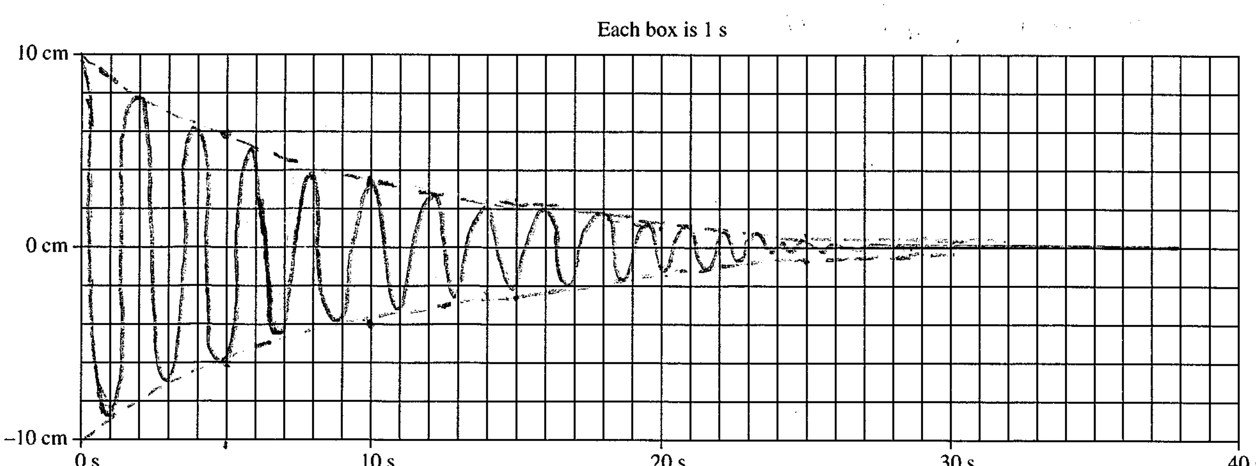
24. A block on a spring oscillates horizontally on a table with friction. Draw and label force vectors on the block to show all *horizontal* forces on the block.
    a. The mass is to the right of the equilibrium point and moving away from it.



    b. The mass is to the right of the equilibrium point and approaching it.



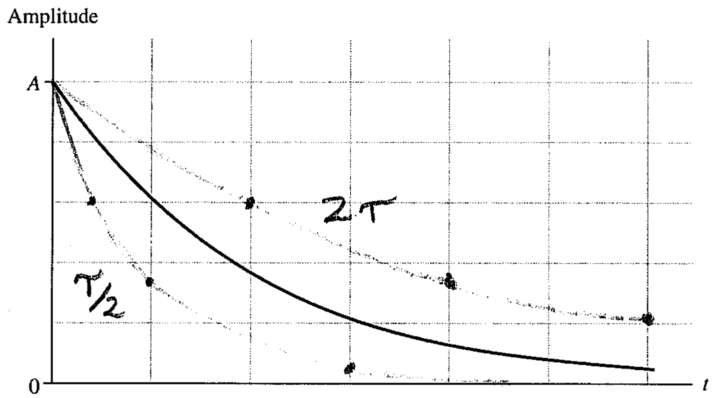
25. A mass oscillating on a spring has a frequency of 0.5 Hz and a damping time constant $\tau = 5$ s. Use the grid below to draw a reasonably accurate position-versus-time graph lasting 40 s.



Envelope of the amplitude: $x_{max} = Ae^{-t/2\tau}$

26. The figure below shows the envelope of the oscillations of a lightly damped oscillator. On the same axes, draw the envelope of oscillations if

a. The time constant is doubled.
b. The time constant is halved.



15.8 Driven Oscillations and Resonance

27. What is the difference between the driving frequency and the natural frequency of an oscillator?

The driving frequency is the frequency of a periodic external force that is applied to a system (an oscillator). The natural frequency is the frequency of the oscillator when its motion is only affected by its restoring force (free of any external forces). The driving frequency and the natural frequency are independent.

28. A car drives along a bumpy road on which the bumps are equally spaced. At a speed of 20 mph, the frequency of hitting bumps is equal to the natural frequency of the car bouncing on its springs.

a. Draw a graph of the car's vertical bouncing amplitude as a function of its speed if the car has new shock absorbers (large damping coefficient).
b. Draw a graph of the car's vertical bouncing amplitude as a function of its speed if the car has worn-out shock absorbers (small damping coefficient).

Draw both graphs on the same axes, and label them as to which is which.