

14 Fluids and Elasticity

14.1 Fluids

1. An object has density ρ .

- a. Suppose each of the object's three dimensions is increased by a factor of 2 without changing the material of which the object is made. Will the density change? If so, by what factor? Explain.

The density will not change. Density is the ratio of mass over volume, and both mass and volume increase by $2^3 = 8$ times.

- b. Suppose each of the object's three dimensions is increased by a factor of 2 without changing the object's mass. Will the density change? If so, by what factor? Explain.

The density will decrease by a factor of 8 (or 2^3).

$$\frac{\rho'}{\rho} = \frac{m'/V'}{m/V} = \frac{m/2^3V}{m/V} = \frac{1}{8} \quad \text{so} \quad \rho' = \frac{1}{8}\rho$$

2. Air enclosed in a cylinder has density $\rho = 1.4 \text{ kg/m}^3$.

- a. What will be the density of the air if the length of the cylinder is doubled while the radius is unchanged?

$$\frac{\rho'}{\rho} = \frac{m'/V'}{m/V} \quad \text{but since the cylinder is enclosed, } m' = m.$$

$$\frac{\rho'}{\rho} = \frac{m/2V}{m/V} = \frac{1}{2} \quad \text{so} \quad \rho' = \frac{1}{2}\rho = \frac{1}{2}(1.4 \frac{\text{kg}}{\text{m}^3}) = 0.7 \frac{\text{kg}}{\text{m}^3}$$

- b. What will be the density of the air if the radius of the cylinder is halved while the length is unchanged?

The cross-sectional area is πr^2 so if the radius is halved the area and volume decrease by a factor of 4. $\rho' = \frac{m'}{V'} = \frac{m}{4V} = \frac{1}{4}(1.4 \frac{\text{kg}}{\text{m}^3}) = 0.35 \frac{\text{kg}}{\text{m}^3}$

3. Air enclosed in a sphere has density $\rho = 1.4 \text{ kg/m}^3$. What will the density be if the radius of the sphere is halved?

Volume(V) $\propto r^3$ so if $r' = r/2$ then

$$\rho' = \frac{m'}{V'} = \frac{m}{V/8} = 8\rho = 8(1.4 \frac{\text{kg}}{\text{m}^3}) = 11.2 \frac{\text{kg}}{\text{m}^3}$$

14.2 Pressure

14.3 Measuring and Using Pressure

4. When you stand on a bathroom scale, it reads 700 N. Suppose a giant vacuum cleaner sucks half the air out of the room, reducing the pressure to 0.5 atm. Would the scale reading increase, decrease, or stay the same? Explain.

The scale reading stays the same. Pressure is a scalar, exerting no net upward or downward force on the scale.

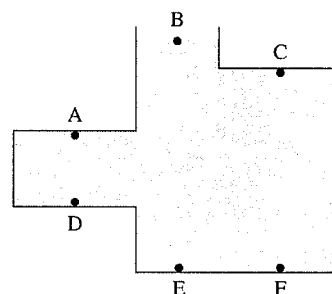
5. Rank in order, from largest to smallest, the pressures at A, B, and C.

Order: $A > C > B$

Explanation:

The pressure depends on the depth (h) below the surface (or opening). $h_A > h_C > h_B = 0$

Note: $h > 0$ when below the surface.



6. Refer to the figure in Exercise 5. Rank in order, from largest to smallest, the pressures at D, E, and F.

Order: $E = F > D$

Explanation:

Pressure $\propto h$ (depth below surface)

$$h_E = h_F > h_D$$

7. The gauge pressure at the bottom of a cylinder of liquid is $p_g = 0.4$ atm. The liquid is poured into another cylinder with twice the radius of the first cylinder. What is the gauge pressure at the bottom of the second cylinder?

$$p_g = \rho g h = \rho g \frac{V}{A} \text{ where both cylinders have the same volume } (V), \text{ but different cross-sectional areas: } A_2 = \pi r_2^2 = \pi (2r_1)^2 = 4\pi r_1^2 = 4A_1$$

so

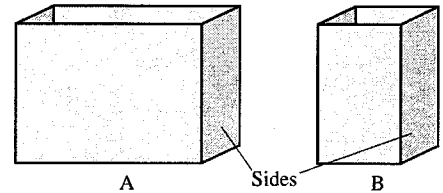
$$p_{g2} = \rho g \frac{V}{A_2} = \rho g \frac{V}{4A_1} = \frac{1}{4} p_{g1} = \frac{1}{4} (0.4 \text{ atm}) = 0.1 \text{ atm}$$

8. Cylinders A and B contain liquids. The pressure p_A at the bottom of A is higher than the pressure p_B at the bottom of B. Is the ratio p_A/p_B of the absolute pressures larger than, smaller than, or equal to the ratio of the gauge pressures? Explain.

The ratio of absolute pressures is smaller. The absolute pressure is gauge pressure plus atmospheric pressure.

$$\frac{P_{gA} + P_{atm}}{P_{gB} + P_{atm}} < \frac{P_{gA}}{P_{gB}}$$

9. A and B are rectangular tanks full of water. They have equal depths, equal thicknesses (the dimension into the page), but different widths.



- a. Compare the forces the water exerts on the bottoms of the tanks. Is F_A larger than, smaller than, or equal to F_B ? Explain.

The pressures are equal at the bottom, but F_A is larger than F_B since F is the product of water pressure and the area of a tank's bottom. The bottom area of tank A is larger.

- b. Compare the forces the water exerts on the sides of the tanks. Is F_A larger than, smaller than, or equal to F_B ? Explain.

$F_A = F_B$. The areas of the given sides are the same and the water pressure at any given depth is the same for both A and B.

10. Water expands when heated. Suppose a beaker of water is heated from 10°C to 90°C . Does the pressure at the bottom of the beaker increase, decrease, or stay the same? Explain.

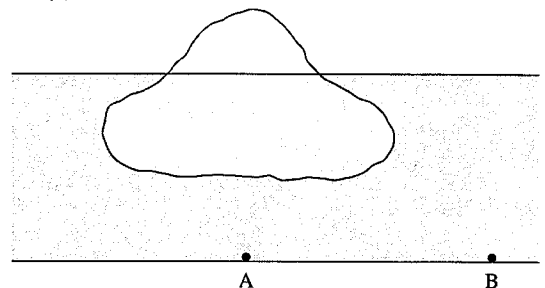
The pressure stays the same. $P = \rho gh + P_{atm}$
The depth (h) increases by the same factor that the density (ρ) of the water decreases, where

$$\rho = \frac{m}{V} = \frac{m}{(\pi r^2) h} \quad \rho \propto \frac{1}{h}$$

11. Is p_A larger than, smaller than, or equal to p_B ? Explain.

$$P_A = P_B = \rho gh + P_{atm}$$

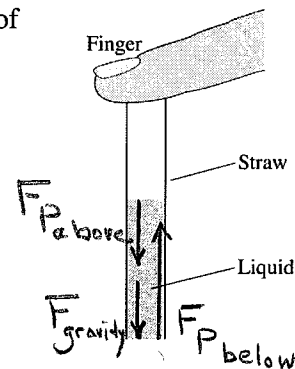
The pressure is distributed throughout the fluid to be equal at equal depths.



12. It is well known that you can trap liquid in a drinking straw by placing the tip of your finger over the top while the straw is in the liquid, and then lifting it out. The liquid runs out when you release your finger.

a. What is the *net* force on the cylinder of trapped liquid?

Zero since the trapped liquid is at rest (not accelerating).



- b. Three forces act on the trapped liquid. Draw and label all three on the figure.
- c. Is the gas pressure inside the straw, between the liquid and your finger, greater than, less than, or equal to atmospheric pressure? Explain, basing your explanation on your answers to parts a and b.

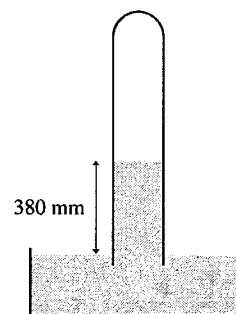
The gas pressure inside the straw is less than atmospheric pressure. The liquid outside of the straw (and below the trapped liquid) is exposed to atmospheric pressure at the surface, say 1 atm. Since the trapped liquid has no net force on it, the pressure just below the trapped liquid (at 1 atm) must be the same as the sum of the pressure from the trapped gas ($< 1 \text{ atm}$) and pressure from F_{gravity} of trapped liquid.

- d. If your answer to part c was “greater” or “less,” how did the pressure change from the atmospheric pressure that was present when you placed your finger over the top of the straw?

The trapped liquid drops slightly before reaching equilibrium (zero net force). The trapped air now occupies a larger volume reducing the pressure inside to a value less than atmospheric pressure.

13. At sea level, the height of the mercury column in a sealed glass tube is 380 mm. What can you say about the contents of the space above the mercury? Be as specific as you can.

The height of the mercury column is half of what it would have been if the space above were completely evacuated. Thus, the space is under a pressure of 0.5 atm. ($1 \text{ atm} = 760 \text{ mm Hg}$)



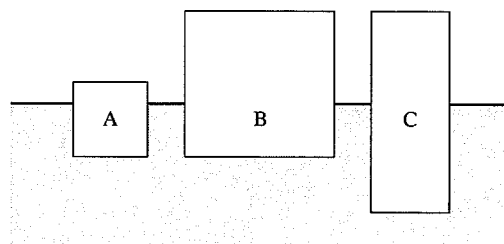
14.4 Buoyancy

14. Rank in order, from largest to smallest, the densities of A, B, and C.

Order: $A > B > C$

Explanation:

A higher density floating object will have a smaller fraction of its volume floating above the fluid level.

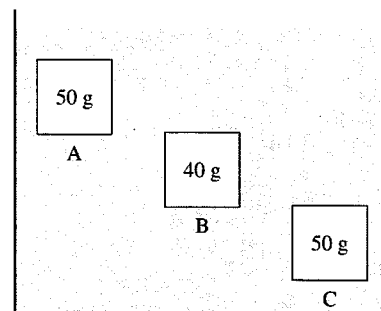


15. A, B, and C have the same volume. Rank in order, from largest to smallest, the sizes of the buoyant forces F_A , F_B , and F_C on A, B, and C.

Order: $F_A = F_B = F_C$

Explanation:

Since they have the same volume, they each displace the same amount of fluid and experience the same upward buoyant force.



16. Refer to the figure of Exercise 15. Now A, B, and C have the same density. Rank in order, from largest to smallest, the sizes of the buoyant forces on A, B, and C.

Order: $F_A = F_C > F_B$

Explanation:

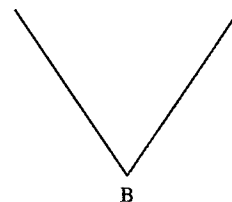
$\rho = \frac{m}{V}$ or $V = \frac{m}{\rho}$ where $\rho_A = \rho_B = \rho_C$ and where $m_A = m_C > m_B$. So $V_B < V_A = V_C$. B displaces the smallest volume of fluid so the upward buoyant force on B is the smallest.

17. Suppose you stand on a bathroom scale that is on the bottom of a swimming pool. The water comes up to your waist.

Does the scale correctly read your weight mg ? If not, does the scale read more than or less than your weight? Explain.

The scale reads a value that is less than mg since the normal force on the scale (which gives the scale reading) is now reduced by an amount equal to the upward buoyant force acting on you.

18. Ships A and B have the same height and the same mass. Their cross-section profiles are shown in the figure. Does one ship ride higher in the water (more height above the water line) than the other? If so, which one? Explain.



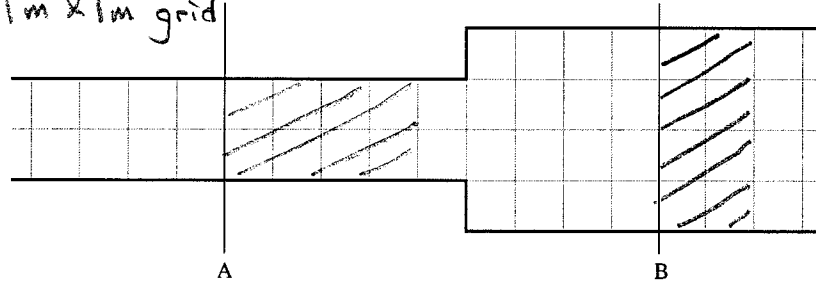
Ship A rides higher than ship B since B must ride lower in the water to displace the same volume of water as A.

The displaced volume of water and the buoyant force must be the same for both to balance out the force of gravity acting on each ship.

14.5 Fluid Dynamics

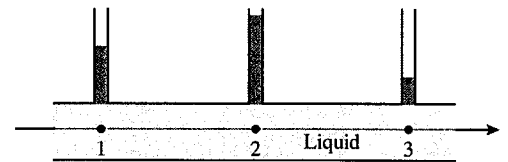
19. A stream flows from left to right through the constant-depth channel shown below in an overhead view. A grid has been added to facilitate measurement. The fluid's flow speed at A is 2 m/s.

Assume 1m x 1m grid



- Shade in squares to represent the water that has flowed past point A in the last two seconds.
 - Shade in squares to represent the water that has flowed past point B in the last two seconds.
20. Can the following be reasonably modeled as ideal fluids? Answer Yes or No.
- Water flowing through a tube. Yes
 - Honey flowing through a tube. No.
 - A whitewater river. No.
 - Water flowing slowing through a concrete-lined channel. Yes.

21. Liquid flows through a pipe. You can't see into the pipe to know how the inner diameter changes. Rank in order, from largest to smallest, the flow speeds v_1 to v_3 at points 1, 2, and 3.

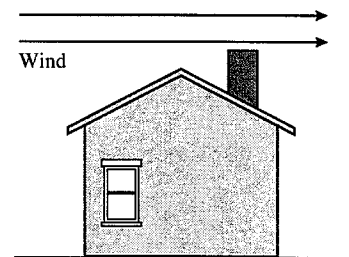


Order: $v_3 > v_1 > v_2$

Explanation:

As the speed of the fluid increases, the pressure is reduced.

22. Wind blows over a house. A window on the ground floor is open. Is there an air flow through the house? If so, does the air flow in the window and out the chimney, or in the chimney and out the window? Explain.



The pressure is reduced at the top of the chimney due to the wind (high flow rate). So air flows in the window and out the chimney.

14.6 Elasticity

23. A force stretches a wire by 1 mm.

- a. A second wire of the same material has the same cross section and twice the length. How far will it be stretched by the same force? Explain.

The second wire will be stretched by 2mm because the amount of stretching is proportional to the length of the wire. $\frac{F/A}{\Delta L/L} = Y$, $\Delta L = \frac{F}{A} \frac{L}{Y}$ so $\Delta L' = \frac{F}{A} \frac{2L}{Y} = 2\Delta L$

- b. A third wire of the same material has the same length and twice the diameter as the first. How far will it be stretched by the same force? Explain.

Because the cross-sectional area is increased by 4 times when the diameter is doubled, the stretching will be $1/4$ as much or $\Delta L' = \frac{F}{4A} \frac{L}{Y} = \frac{\Delta L}{4} = 0.25 \text{ mm}$.

24. A 2000 N force stretches a wire by 1 mm.

- a. A second wire of the same material is twice as long and has twice the diameter. How much force is needed to stretch it by 1 mm? Explain.

Because the cross-sectional area increases by 4 times, but length only doubles, a force of $2(2000\text{N}) = 4000\text{N}$ is required. $F = AY \frac{\Delta L}{L}$, $F' = 4AY \frac{\Delta L}{2L} = 2F$.

- b. A third wire is twice as long as the first and has the same diameter. How far is it stretched by a 4000 N force?

It is stretched by 4mm.

$$\Delta L' = \frac{2F}{A} \frac{2L}{Y} = 4\Delta L.$$

25. A wire is stretched right to the breaking point by a 5000 N force. A longer wire made of the same material has the same diameter. Is the force that will stretch it right to the breaking point larger than, smaller than, or equal to 5000 N? Explain.

The longer wire will also break at 5000N. The force per area is the same in both cases.

26. Sphere A is compressed by 1% at an ocean depth of 4000 m. Sphere B is compressed by 1% at an ocean depth of 5000 m. Which has the larger bulk modulus? Explain.

Sphere B has a larger bulk modulus since it takes a greater pressure (greater depth) to compress it by same percentage (same $\Delta V/V$).