13 Newton's Theory of Gravity

13.1 A Little History

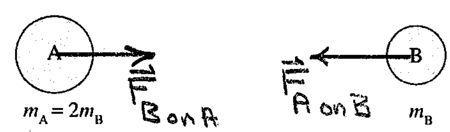
13.2 Isaac Newton

13.3 Newton's Law of Gravity

1. Is the earth's gravitational force on the moon larger than, smaller than, or equal to the moon's gravitational force on the earth? Explain.

By Newton's 3rd Law of Motion the gravitational forces are equal and opposite.

2. Star A is twice as massive as star B. They attract each other.



   a. Draw gravitational force vectors on both stars. The length of each vector should be proportional to the size of the force. Equal Lengths.

   b. Is the acceleration of star A larger than, smaller than, or equal to the acceleration of star B? Explain.

The acceleration of star A is smaller. For equal forces, the larger mass experiences the smaller acceleration because $a = F/m$.

3. The gravitational force of a star on orbiting planet 1 is F_1. Planet 2, which is twice as massive as planet 1 and orbits at half the distance from the star, experiences gravitational force F_2. What is the ratio F_2/F_1?

Size of $F_1 = \dfrac{GMm_1}{r_1^2}$. Size of $F_2 = \dfrac{GM(2m_1)}{(r_1/2)^2}$.

$$\frac{F_2}{F_1} = \frac{\dfrac{GM(2m_1)}{(r_1/2)^2}}{\dfrac{GMm_1}{r_1^2}} = \frac{2}{(1/2)^2} = 8$$

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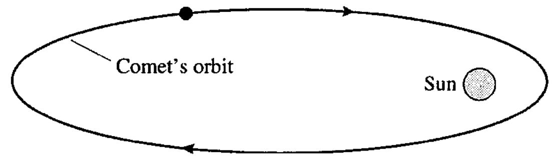
4. Comets orbit the sun in highly elliptical orbits. A new comet is sighted at time t_1.



a. Later, at time t_2, the comet's acceleration a_2 is twice as large as the acceleration a_1 it had at t_1. What is the ratio r_2/r_1 of the comet's distance from the sun at t_2 to its distance at t_1?

Assuming the comet's mass is the same at t_1 and t_2, then $a_2 = 2a_1$ gives $F_2 = 2F_1$

and $\frac{F_2}{F_1} = \frac{(1/r_2)^2}{(1/r_1)^2} = \frac{r_1^2}{r_2^2} = 2$ so $\frac{r_2}{r_1} = \frac{1}{\sqrt{2}}$

b. Still later, at time t_3, the comet has rounded the sun and is headed back out to the farthest reaches of the solar system. The size of the force F_3 on the comet at t_3 is the same as the size of force F_2 at t_2, but the comet's distance from the sun r_3 is only 90% of distance r_2. Astronomers recognize that the comet has lost mass. Part of it was "boiled away" by the heat of the sun during the time of closest approach, thus forming the comet's tail. What percent of its initial mass did the comet lose?

$F_3 \propto \frac{m_3}{r_3^2} = \frac{m_2}{r_2^2}$ so $m_3 = \frac{r_3^2}{r_2^2} m_2$.

Now $r_3 = 0.90\, r_2$ gives $m_3 = \frac{(0.90\, r_2)^2}{r_2^2} m_2$.

and $m_3 = 0.81\, m_2$. $\boxed{\text{So comet lost 19\% of its initial mass.}}$

13.4 Little *g* and Big *G*

5. How far away from the earth does an orbiting spacecraft have to be in order for the astronauts inside to be weightless?

The astronauts can be weightless at any distance because an object is said to be weightless if it is in freefall (as in orbit). For the gravitational force to become zero, the spacecraft would have to be an infinite distance away.

6. The free-fall acceleration at the surface of planet 1 is 20 m/s^2. The radius and the mass of planet 2 are half those of planet 1. What is g on planet 2?

$g \propto \frac{m}{r^2}$. So $g_2 = g_1 \frac{1/2}{(1/2)^2} = 2g_1$.

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13.5 Gravitational Potential Energy

7. Explain *why* the gravitational potential energy of two masses is negative. Note that saying "because that's what the formula gives" is *not* an explanation. An explanation makes use of the basic ideas of force and potential energy.

The gravitational potential energy is negative because we choose to place the zero point of potential energy at infinity (i.e., the separation of masses is infinite). This choice with the conservative gravitational force being attractive gives negative $U_{gravity}$ for finite mass separation. As the masses approach each other $U_{gravity}$ becomes more negative as K increases.

13.6 Satellite Orbits and Energies

8. Planet X orbits the star Alpha with a "year" that is 200 earth days long. Planet Y circles Alpha at nine times the distance of planet X. How long is a year on planet Y?

From Kepler's 3rd Law, the orbital period squared is proportional to the orbital radius cubed: $T^2 \propto r^3$

So with $r_y = 9r_x$ we have

$$\frac{T_y^2}{T_x^2} = \frac{r_y^3}{r_x^3} = \frac{(9r_x)^3}{r_x^3} \quad \text{so} \quad T_y = \sqrt{9^3}\, T_x = 27 T_x$$

gives $T_y = 27(200 \text{ days}) = 5400 \text{ days}$

9. The mass of Jupiter is $M_{\text{Jupiter}} = 300M_{\text{earth}}$. Jupiter orbits around the sun with $T_{\text{Jupiter}} = 11.9$ years in an orbit with $r_{\text{Jupiter}} = 5.2r_{\text{earth}}$. Suppose the earth could be moved to the distance of Jupiter and placed in a circular orbit around the sun. The new period of the earth's orbit would be

a. 1 year.
b. 11.9 years.
c. Between 1 year and 11.9 years.
d. More than 11.9 years.
e. It could be anything, depending on the speed the earth is given.
f. It is impossible for a planet of earth's mass to orbit at the distance of Jupiter.

Circle the letter of the true statement. Then explain your choice.

The orbital period is independent of the mass of the orbiting body, provided that the orbiting body's mass is much less than the mass of the body being orbited.

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10. Satellite A orbits a planet with a speed of 10,000 m/s. Satellite B is twice as massive as satellite A and orbits at twice the distance from the center of the planet. What is the speed of satellite B?

The orbital speed (v) is independent of the orbiting mass but depends on the distance (r) from the planet's center as $1/\sqrt{r}$. Given: $r_B = 2r_A$

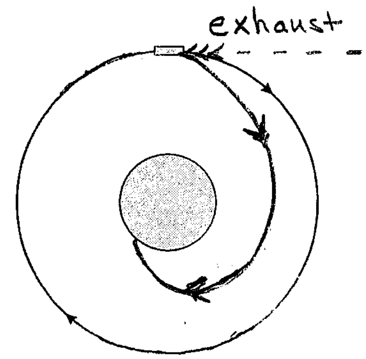
$$\frac{v_B}{v_A} = \frac{1/\sqrt{r_B}}{1/\sqrt{r_A}} = \frac{1/\sqrt{2r_A}}{1/\sqrt{r_A}} = \frac{1}{\sqrt{2}}$$

$$v_B = \frac{1}{\sqrt{2}} v_A = \frac{1}{\sqrt{2}}(10{,}000 \text{ m/s}) \approx 7100 \text{ m/s}$$

11. a. A crew of a spacecraft in a clockwise circular orbit around the moon wants to change to a new orbit that will take them down to the surface. In which direction should they fire the rocket engine? On the figure, show the exhaust gases coming out of the spacecraft.

b. On the figure, show the spacecraft's orbit after firing its rocket engine.

c. The moon has no atmosphere, so the spacecraft will continue unimpeded along its new orbit until either firing its rocket again or (ouch!) intersecting the surface. As it descends, does its speed increase, decrease, or stay the same? Explain.



The initial firing of the rocket engine exerts a force opposite the velocity of the spacecraft in its circular orbit. During the short time this force exists, the spacecraft slows down enough to enter a new orbit that follows a spiral path to the moon. As the spacecraft descends its speed increases as a decrease in its gravitational potential energy gives an increase in its kinetic energy.

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