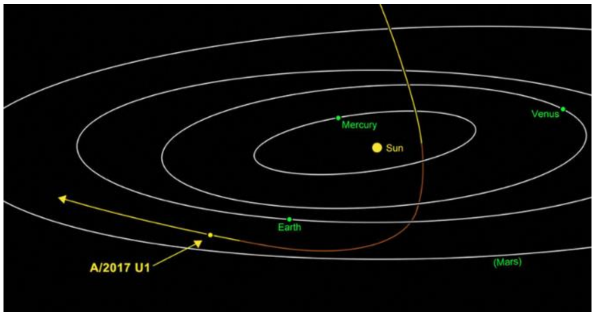
Ph 212 In-Class Exercise – Oumuamua

In 2017 a visitor from outside the Solar System was spotted flying through our planetary system passing not too far from Earth.



This likely galactic asteroid was named Oumuamua.

Oumuamua’s trip through the Solar System was so fast that there was not chance that we could send a probe to visit it. All we got was some visual data that allowed to infer that Oumuamua was long and thin in shape. By some estimates [1] the object might be 1000 meters long by 35 meters by 35 meters wide. No mass measurements have been able yet to be made, but it is not unreasonable to estimate that such an object might have a mass of around 6 billion kilograms.

If some day we are ever able to visit such an object with a probe, it will be important to understand the gravitational field around such a differently shaped object than what we are used to.

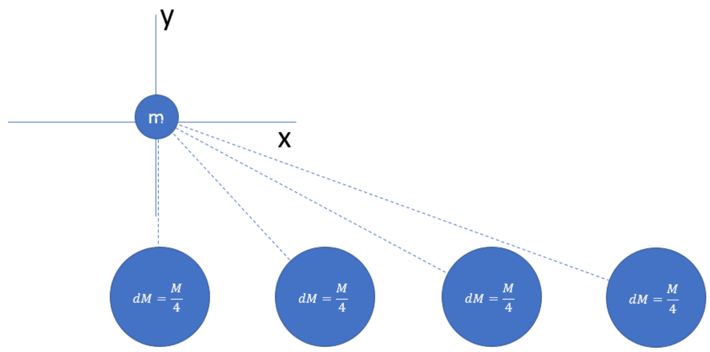
Goal:
Find the gravitational acceleration of a small space probe of mass **m** a distance **h** above one side of a long thin space rock of length **L** and mass **M**.

[1] Cofield, Calia (14 November 2018). "NASA Learns More About Interstellar Visitor 'Oumuamua"

Step 1:
For point-sized or spherical objects, the gravitational force of one object upon another is described by the equation:

$$\vec{F} = \frac{GmM}{s^2}\hat{s}$$

In order to estimate the effects of gravity at the point of the space probe for an object in the shape of Oumuamua, let's approximate the Oumuamua as 4 point-masses each with 1/4th the mass of Oumuamua as shown:



Note that the mass of each piece is M/4.
It would be useful for you to call each of these pieces dm where dm = M/4.

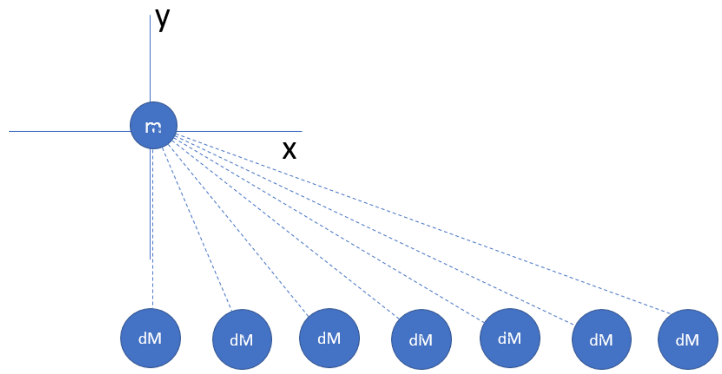
Using a little bit of trigonometry and the tools we learned in Ph 211, we can draw a FBD from the probe and find the Force upon and the acceleration of the probe due to Oumuamua.

Use a calculator or excel or python to do this calculation. For the position and mass of the probe use h = 10 meters and m = 1000 kg.

Questions for part 1:
Did you use a calculator, excel or python to do this calculation?
Explain why you made this choice.

What value did you get for Fnet upon the probe?

Step 2:
The value for g that found in Step 1 was only an approximation. A better approximation would be if you broke Oumuamua into 7 pieces each with $1/7^{th}$ the mass of the whole. Try doing this calculation and see how much your total differs from your number in Step 1.



Questions 2a:
Did you use a calculator, excel or python to do this calculation? Explain why you made this choice.

What value did you get for Fnet upon the probe in this case?

Step 3

In Step 1 we found an approximation to the acceleration of a probe due to the gravity of Oumuamua by breaking Oumuamua up into 4 point like pieces. In Step 2 we found a better approximation by breaking Oumuamua into 7 pieces. If you are so inclined, you could break up Oumuamua into 15 or 30 or a hundred pieces. But, since you know calculus, it will probably be easier for you to get to the logical conclusion of this process and break Oumuamua into infinitely many tiny pieces.

Thus, instead of using a calculator, excel or python, let's try doing this integral on paper.

Step 3a:
Draw a picture of Oumuamua and your probe. Clearly label these things on your picture:

- L, M and h
- An origin located at the location of the probe
- at least one dm and the s associated with that dm
- a vector representing a rough idea of what Fnet on the probe might look like

Step 3b:
Draw a FBD (including an "= Fnet") for the probe. Make sure that your FBD includes axes.

Step 3c:
Compare your picture and FBD with this picture and FBD.

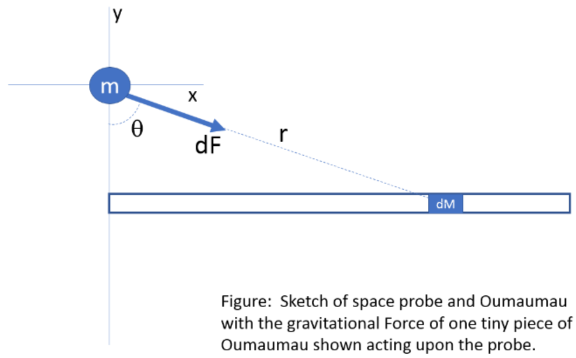
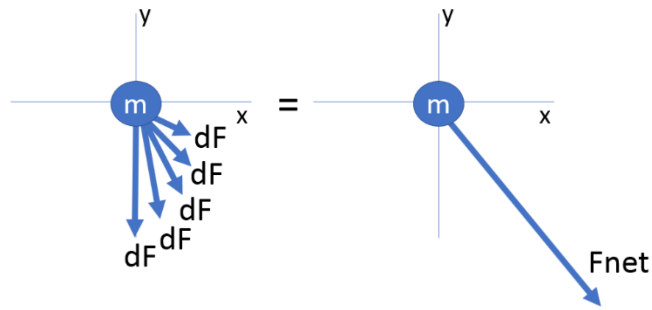


Figure: Sketch of space probe and Oumaumau with the gravitational Force of one tiny piece of Oumaumau shown acting upon the probe.

Free-Body Diagram of Space Probe



In order for us to find Fnet on the probe, we will need to sum up all the tiny dF's acting upon the probe from all the dM's of Oumuamua.

Our goal is thus to create two integrals that find the two components of Fnet upon the probe. Remember, the gravitational attraction between a point mass and the probe is describe by

$$\vec{F} = \frac{GmM}{s^2}\hat{s}\,.$$

Here is some help getting started:

$$F_{net-x} = \int \frac{Gm(dM)}{r^2}\sin\theta$$

Now, find all the below expressions in terms Cartesian coordinates:

dM =

r =

sinθ =

Once you figure out what dM, s and sinθ are in this case, update the above expression for F_{net-x} making sure that you include appropriate limits for the definite integral you will need to solve.

Rewrite the full integral in Cartesian coordinates:

$$F_{net-y} = \int \frac{Gm(dM)}{r^2}\cos\theta$$

Now, find all the below expressions in terms Cartesian coordinates:

dM =

r =

cosθ =

Once you figure out what dM, s and cosθ are in this case, update the above expression for F_{net-x} making sure that you include appropriate limits for the definite integral you will need to solve.

Rewrite the full integral in Cartesian coordinates:

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Step 4: You can now either solve by hand or use the integral table on the screen and solve the integrals on the previous page. Do that here.

Question 4:
Plug in numbers and compare your result to your two prior approximations. Show all those results here.

Step 5: Good job! You made it this far. Are you still looking for a project to do? If so, a good Ph 212 project would be to try to model the motion of the small space problem in the problem using VPython. Let your instructor know if you'd like to do this project.