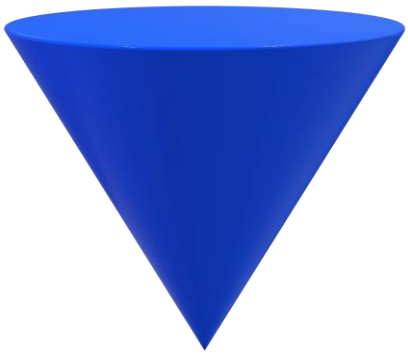
Ph 212 – Rotating a Cone Winter 2021

Goal: Find the mass, center of mass, moment of inertia, KE and L of a cone.



This cone has a height of H and a maximum radius of R and a uniform density of ρ and is centered around the z axis.

Exercise 1: Find the total mass of the cone.
Step 1a: Write down an integral that says that the total mass of the cone is the sum of the masses of a whole bunch of infinitesimal pieces of the cone.

Step 1b: Convert the integral from Step 1a into an integral that says you are going to sum up the mass of a whole bunch thin disks.

Step 1c: The equation you created in Step 1c should have a z and an r in it. State the r in terms of a z.

Step 1d: Perform the integral you derived in Step 1c to find the mass of a cone of height H and maximum radius R.

Exercise 2: Center of Mass

The equation for the z-coordinate of the center of mass of an object can be expressed as $z_{cm} = \frac{1}{M_{tot}} \int z dm$. Find the center of mass of the cone along the z-axis. Make sure that you perform a reasonableness test on your solution.

Exercise 3 -- Moment of Inertia: The equation for the moment of inertia around the z-axis of an infinitesimal particle a distance r from the z-axis is given by the equation $r^2 dm$.
Find the moment of inertia around the z-axis of this cone.

Exercise 4: You start spinning the cone the z-axis with an angular acceleration of $\alpha(t) = bt^2$. What is the Kinetic Energy of the cone at $t = t_f$?

Exercise 5: You drop a sphere of ice cream of mass M on the top of this spinning solid cone. The ice cream sphere sticks to the cone. What is the new rotational velocity of the cone?

Exercise 6: How much energy went into deforming the ice cream that stuck to the cone?